

Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

Lecture-06

Involute Straight Tooth Spur Gear



Outline of the Lecture

- **Design of Gear Tooth based on beam Strength**
- **Lewis form factor**
- **Formative number of teeth**
 - **Consideration of tooth (generation) accuracy**
 - **Velocity factor**
 - **Wear and lubrication factor**

Strength of gear teeth-Lewis equation

With respect to straight tooth spur gear

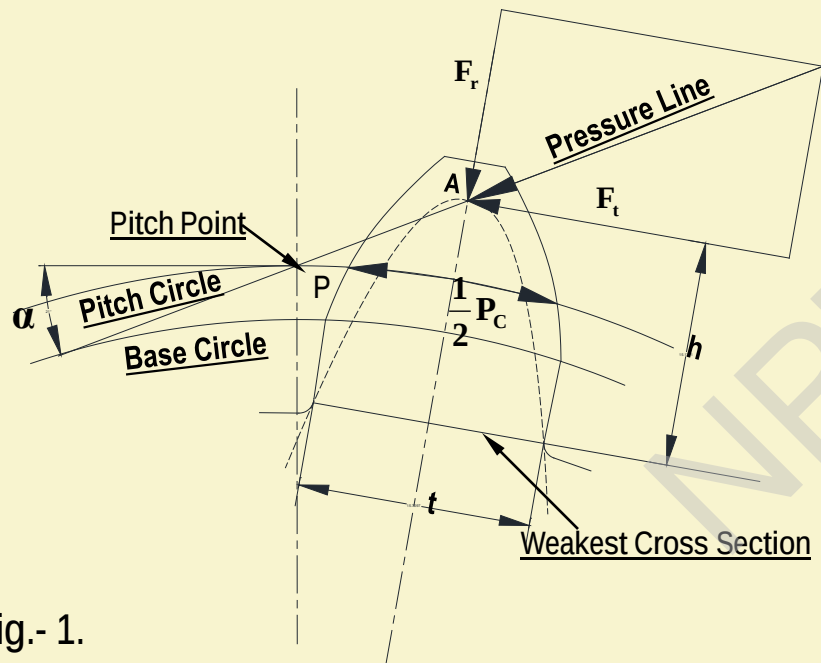


Fig.- 1.

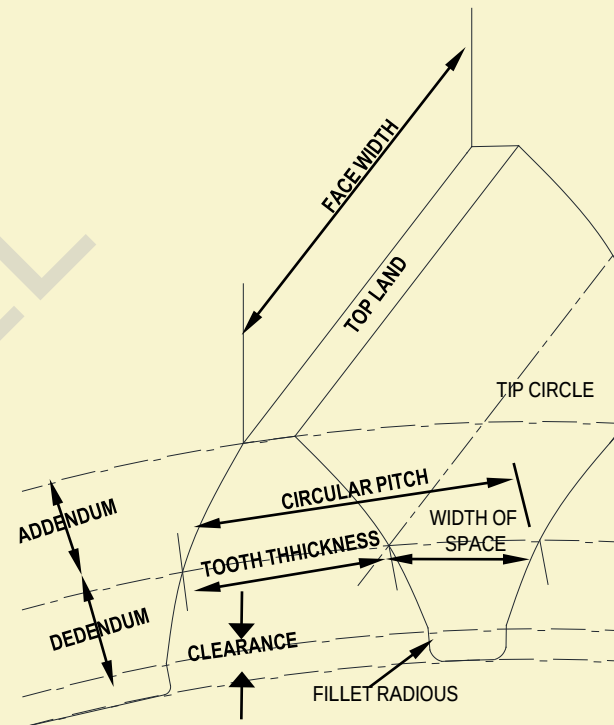


Fig.- 2.

Strength of gear teeth-Lewis equation (Contd....) :

Straight tooth spur gear

Bending Stress at root is derived as:

$$\begin{aligned}\sigma &= \frac{My}{I} = \frac{M(t/2)}{bt^3/12} \\ &= \frac{6M}{bt^2} = \frac{6F_t h}{bt^2}\end{aligned}$$

Where,

M is the bending moment at considered cross-section

I Area moment of inertia

b is the width of gear

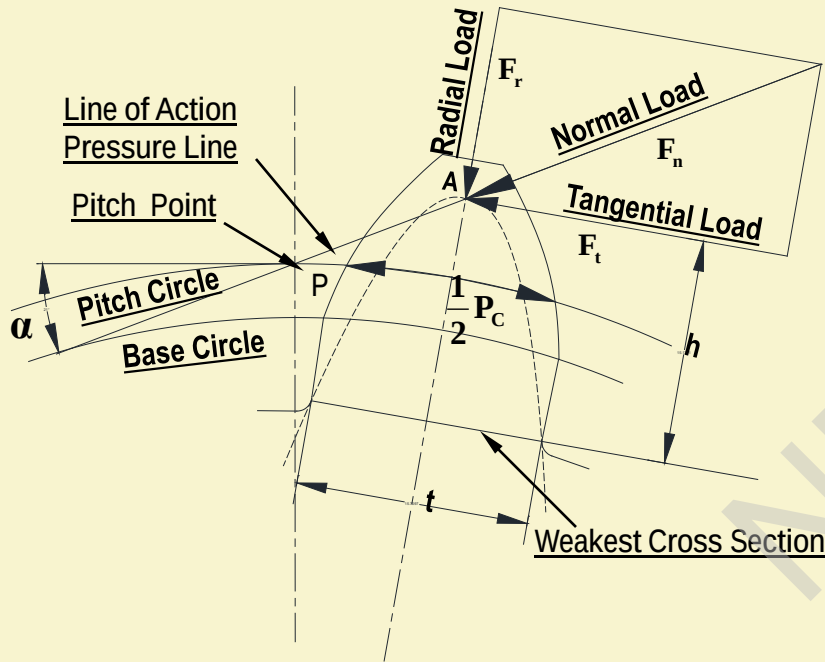
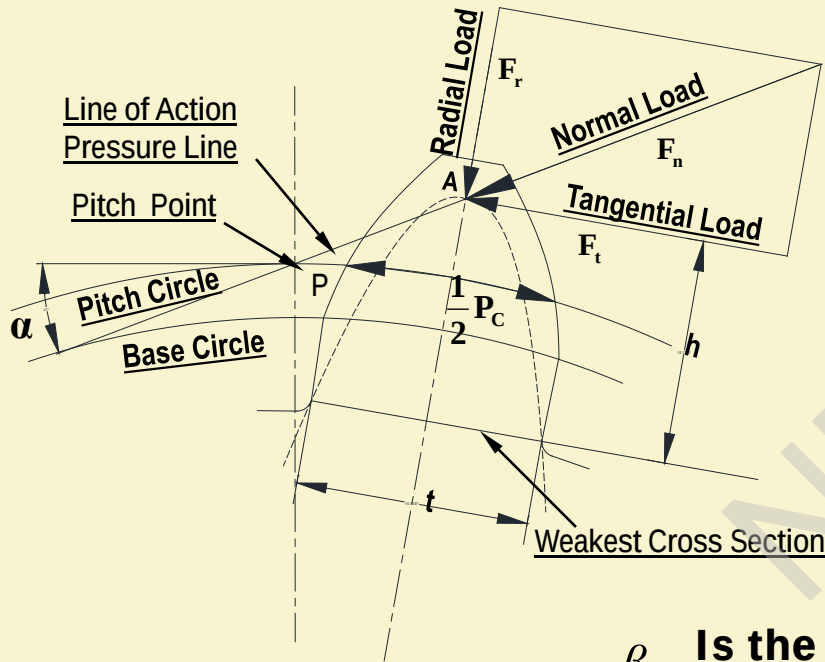


Fig.- 3.

Strength of gear teeth-Lewis equation (Contd....) :

Straight tooth spur gear



Now,
$$\frac{t^2}{6h} = y(\pi m_n) = Ym_n$$

y is called the Lewis form factor

$Y (= \pi y)$ is Modified Lewis form factor

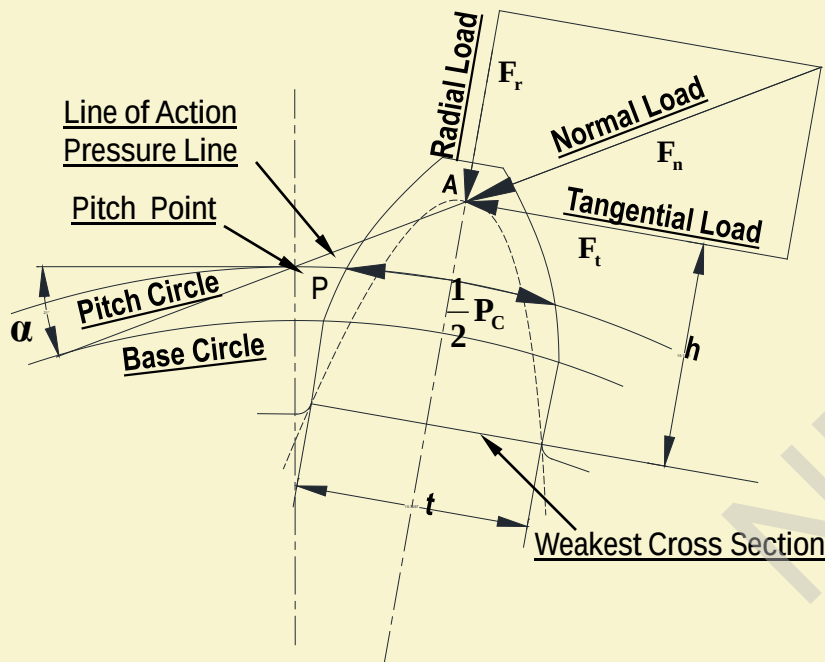
It is expressed as: $Y = 0.484 - 3.28 / Z'$

$Z' = Z / \cos^3(\beta)$ Formative number of teeth.

β Is the helix angle, which is zero in case of straight tooth spur gear.

Strength of gear teeth-Lewis equation (Contd....) :

Straight tooth spur gear



Rearranging:

$$\sigma = \frac{6F_t h}{bt^2}$$

$$F_t = \sigma b Y m_n$$

Introducing Allowable Strength S_o and velocity factor C_v , the maximum tangential load:

$$F_t = S_o c_v b Y m_n$$

Strength of gear teeth-Lewis equation (Contd....) :

Straight tooth spur gear

Velocity Factor C_v is expressed as:

$$c_v = \frac{3}{3 + V}$$

Milled Gear,

$$c_v = \frac{4.5}{4.5 + V}$$

Accurately cut Gear,

$$c_v = \frac{6}{6 + V}$$

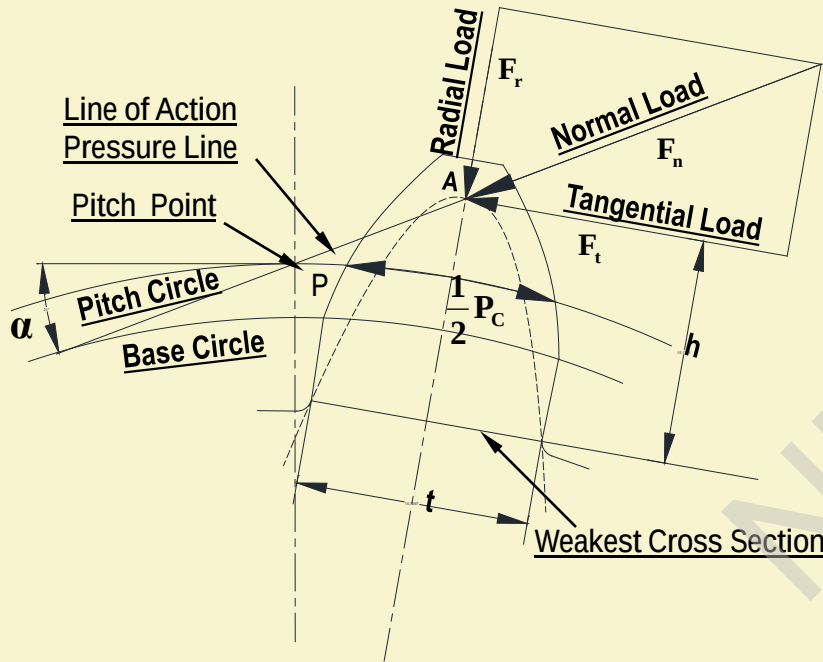
More Accurately cut Gear,

$$c_v = \frac{3.5}{3.5 + \sqrt{V}}$$

Fine Finished Gear,

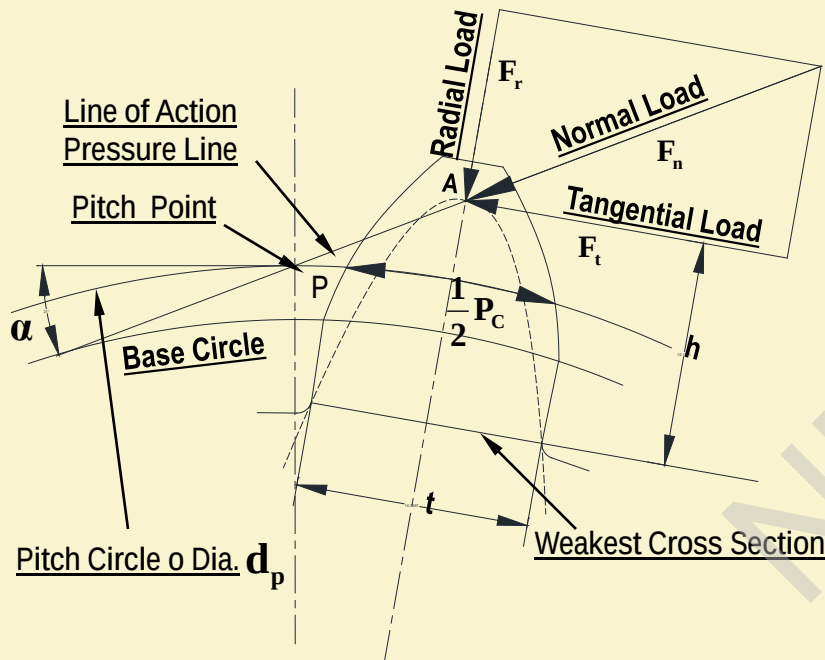
And for Hardened Gear, $c_v = \frac{5.5}{5.5 + \sqrt{V}}$

Where, V is the pitch line velocity of gear in m/sec.



Strength of gear teeth-Lewis equation (Contd....) :

Straight tooth spur gear



Now F_t can be expressed in terms of transmitted Torque T , as:

$$F_t = \frac{2T}{d_p} = \frac{2T}{Zm_n}$$

Also, width b can be limited by a factor ψ multiplied by normal module m_n .

Therefore, equation $F_t = S_o c_v b Y m_n$ can be modified to:

$$T = (S_o c_v Z \psi Y m_n^3) / 2$$

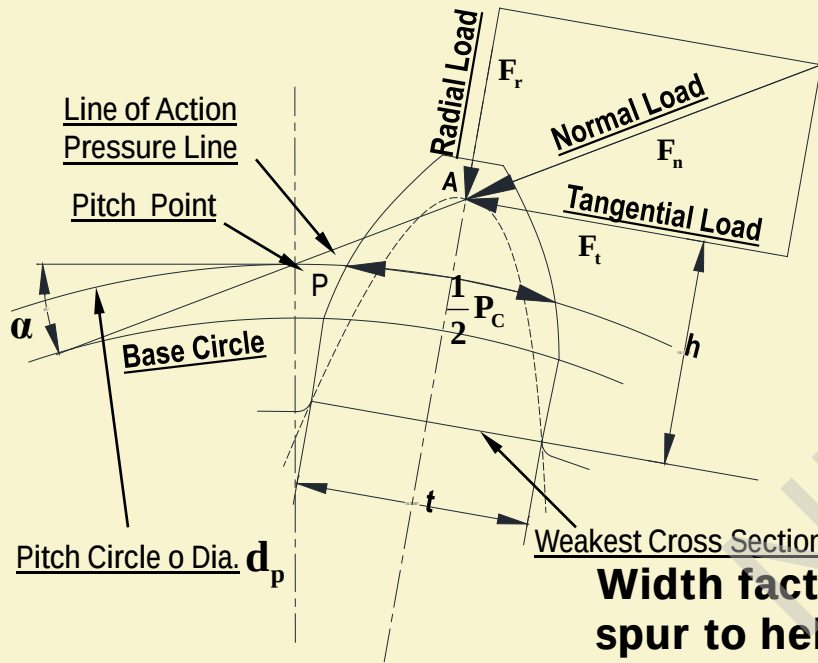
Rearranging:
$$m_n = \sqrt[3]{\frac{2T}{(S_o c_v) Z \psi Y}}$$

Straight tooth spur gear

Therefore, to estimate module:

$$m_n = \sqrt[3]{\frac{2T}{\left(\frac{S_o c_v}{c_w}\right)}} Z_{\psi Y}$$





In the expression:

$$m_n = \sqrt[3]{\frac{2T}{\left(\frac{S_o c_v}{c_w}\right) Z \psi Y}}$$

Lubrication /wear load factor c_w is taken between 1 to 1.25 for force lubrication to splash lubrication respectively.

Width factor ψ may be as high as 15 & 20 for straight tooth spur to helical gear respectively.

$\psi = 10$ to 12 for straight tooth spur and 14 to 16 to helical spur gear is in design common practice.

Thank You



Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

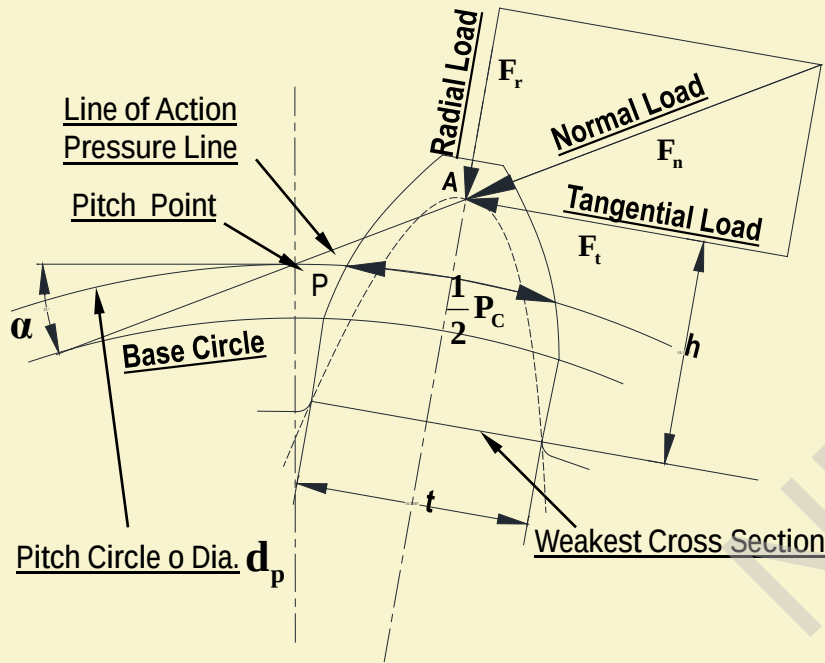
Lecture-07

Helical Tooth Spur Gear & Loads on Gear Shaft

Outline of the Lecture

- **Modification of formula for helical gear (from straight tooth spur gear)**
 - **Equivalent Spur Gear & formative number of teeth**
- **Generalization of formula for designing both straight tooth and helical tooth spur gear**
 - **Pitch circle diameters of gears and pinion, and centre distance**
 - **Loads on shaft through gears**

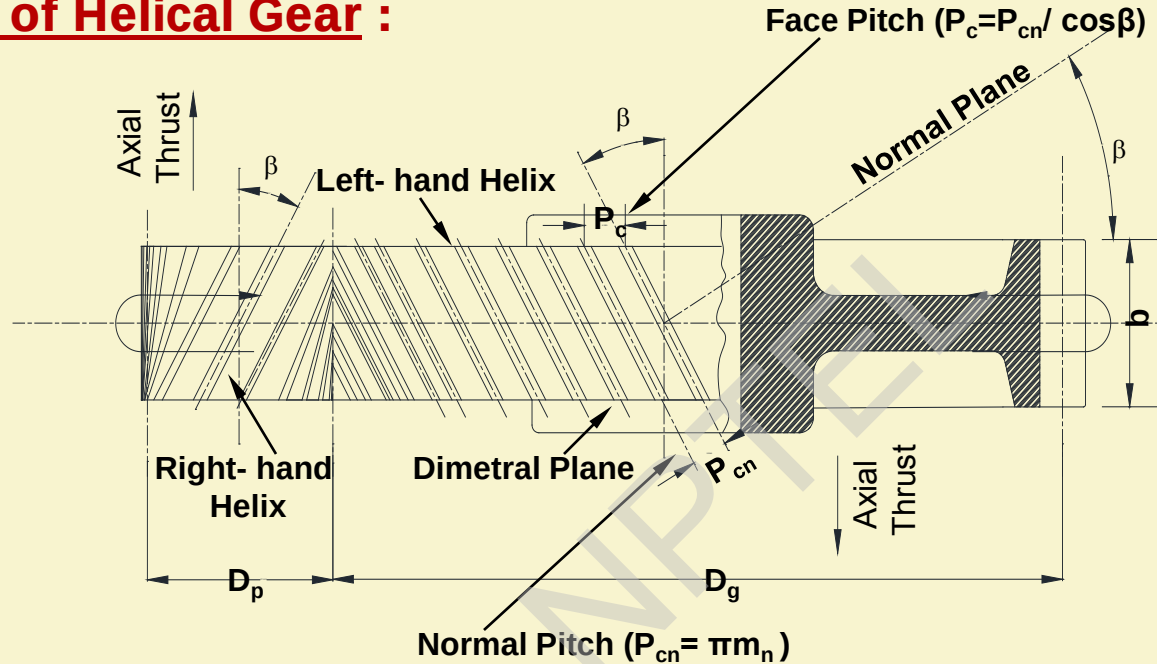
Strength of gear teeth-Lewis equation (Contd...): Straight tooth spur gear (Rpitulation)



To estimate module we use:

$$m_n = \sqrt[3]{\frac{2T}{\left(\frac{S_o c_v}{c_w}\right) Z \psi Y}}$$

Design of Helical Gear :



Plan View of a Helical Gear Pair in Contact

Z' Formative number of teeth is expressed as:

Equivalent Straight tooth spur gear:

$$R = \frac{D}{2 \cos^2 \beta}$$

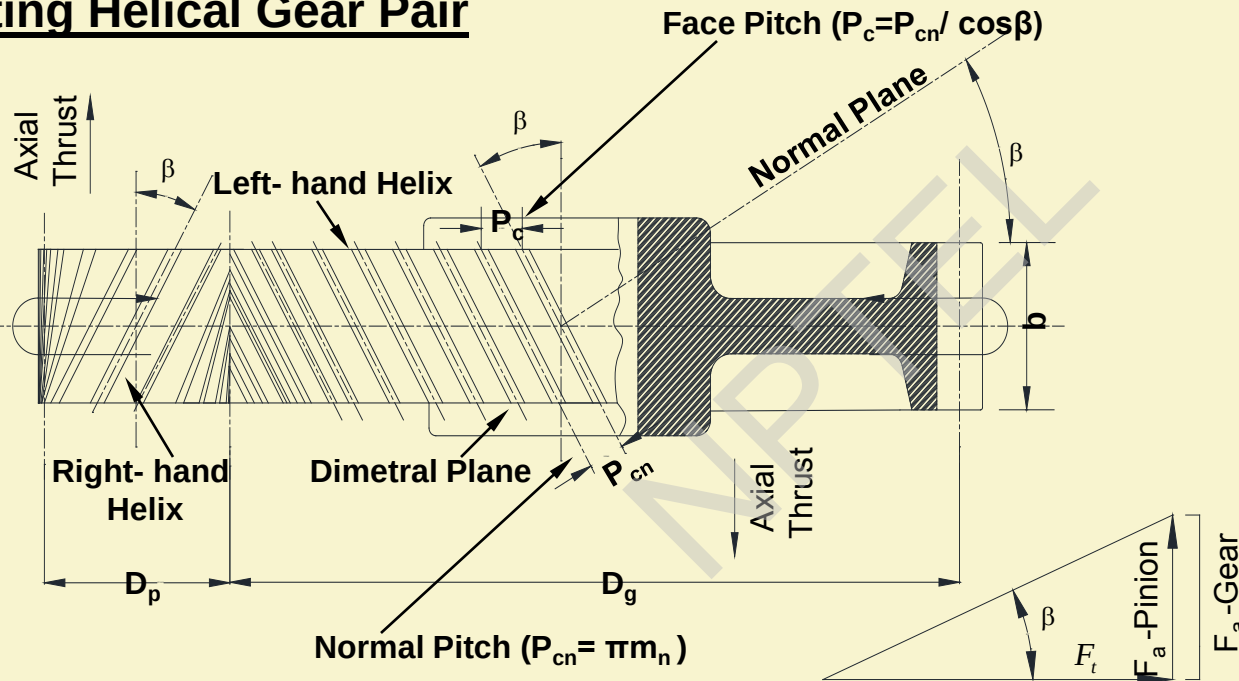
$$2R = \frac{Z m_n / \cos \beta}{\cos^2 \beta}$$

$$Z' m_n = \frac{Z m_n / \cos \beta}{\cos^2 \beta}$$

$$Z' = \frac{Z}{\cos^3 \beta}$$

Design of Helical Gear (Contd...) :

Pitch Diameter of Helical Gear and Centre Distance of Mating Helical Gear Pair



Plan View of a Helical Gear Pair in Contact

$$D_p = \frac{Z_p P_c}{\pi} = \frac{Z_p P_{cn}}{\pi \cos \beta}$$

$$= \frac{Z_p m_n}{\cos \beta}$$

$$D_g = \frac{Z_g P_c}{\pi} = \frac{Z_g P_{cn}}{\pi \cos \beta}$$

$$= \frac{Z_g m_n}{\cos \beta}$$

$$A = (D_p + D_g) / 2$$

$$A = \frac{m_n}{2 \cos \beta} (Z_p + Z_g)$$

Design of Involute Helical Spur Gear

Module on the basis of bending strength:

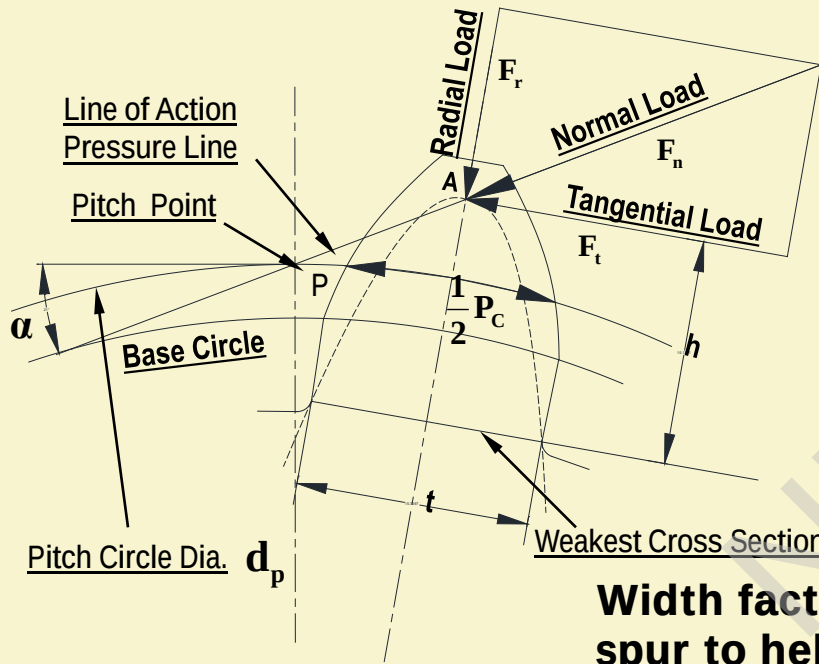
The Lewis Formula for module calculation.

$$m_n = \sqrt[3]{\frac{2T \cos \beta}{S_o C_v \psi YZ C_w}}$$

After selecting module (rounded up to standard) tooth's dynamic load capacity and wear load capacity are verified.

If necessary module is increased.

Strength of gear teeth-Lewis equation (Contd...): Helical tooth spur gear



To estimate module we use:

$$m_n = \sqrt[3]{\frac{2T \cos \beta}{\frac{S_o C_v}{C_w} \psi YZ}}$$

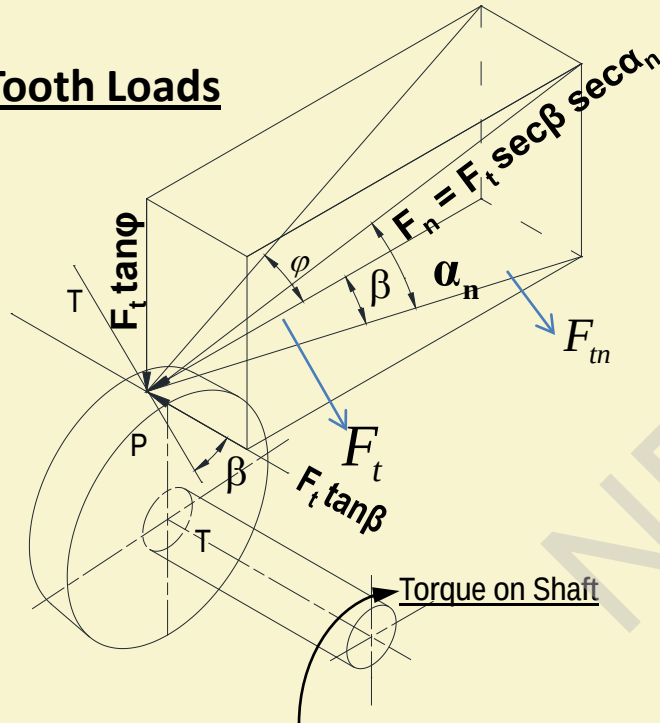
Lubrication /wear load factor C_w is taken between 1 to 1.25 for force lubrication to splash lubrication respectively.

Width factor ψ may be as high as 15 & 20 for straight tooth spur to helical gear respectively.

$\psi = 10$ to 12 for straight tooth spur and 14 to 16 to helical spur gear is in design common practice.

Design of Involute Helical Spur Gear (load on shaft)

Tooth Loads



Forces on Helical Gear Tooth

$$F_t = \frac{2T}{d_p} = \frac{2T \cos \beta}{Zm_n}$$

$$F_{tn} = \frac{F_t}{\cos \beta}$$

$$F_n = \frac{F_{tn}}{\cos \alpha_n} = F_t \sec \beta \sec \alpha_n$$

$$F_r = F_n \sin \alpha_n = F_t \sec \beta \sec \alpha_n \sin \alpha_n$$

$$F_r = F_t \sec \beta \tan \alpha_n = F_t \tan \phi$$

$$F_a = F_n \sin \beta = F_t \tan \beta$$

Thank You



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Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

Lecture-08

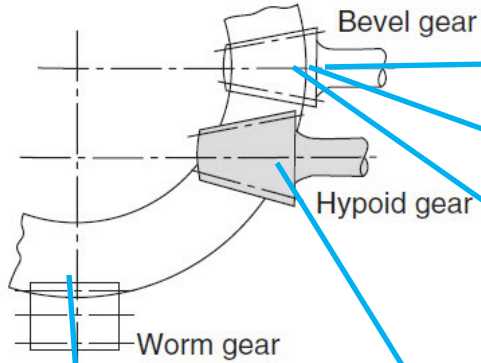
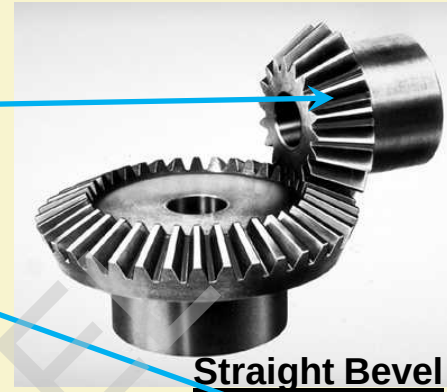
Design of Bevel Gear



Outline of the Lecture

- **Concept and Feature of Bevel Gears**
 - **Nomenclature and Geometry of Straight Bevel Gear**
 - **Formative Number of Teeth of Straight Bevel Gear**
 - **Typical Gear Unit with Bevel and Helical Gears**
 - **Bending Strength basis Design of Straight Bevel Gear**
- **Wear load Capacity and Probable Dynamic Load in Straight Bevel Gear**
- **Tooth Load in Straight Bevel Gear**

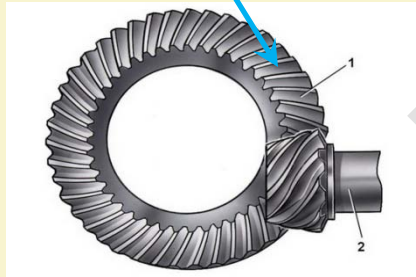
Concept of Bevel Gear



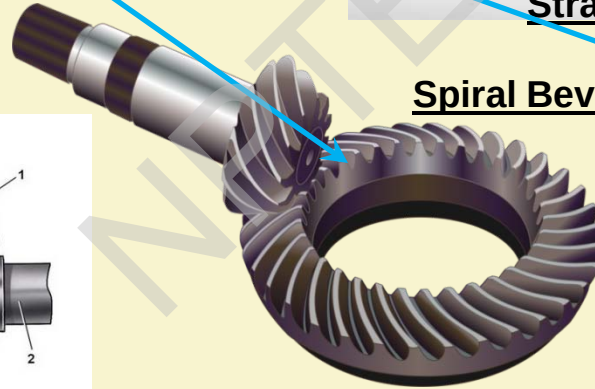
Straight Bevel



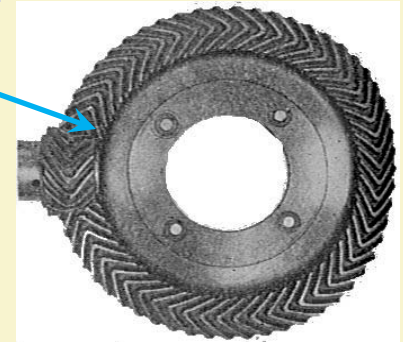
Worm & Worm Wheel



Hypoid Bevel

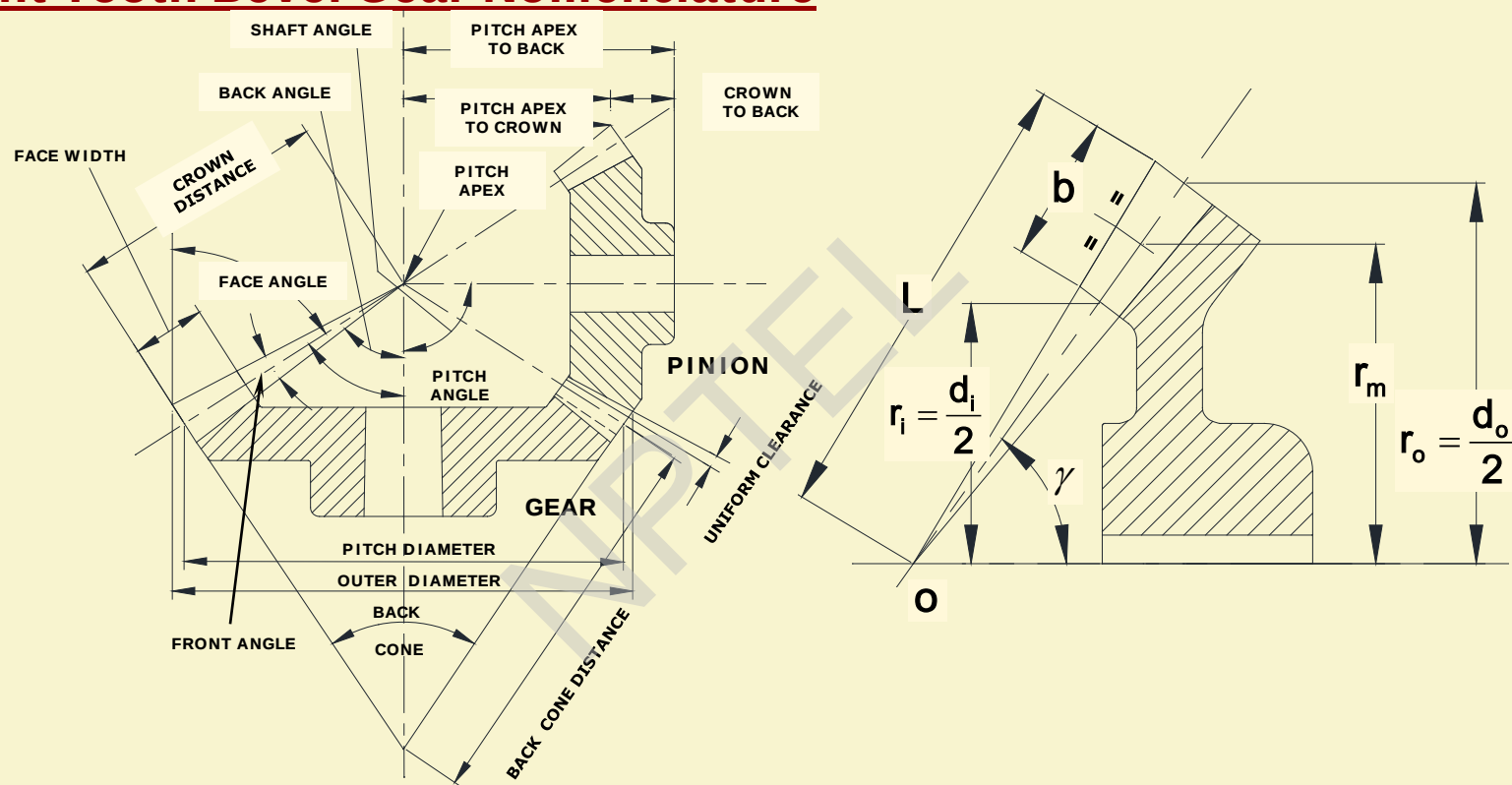


Spiral Bevel



Herringbone Bevel

Straight Tooth Bevel Gear Nomenclature

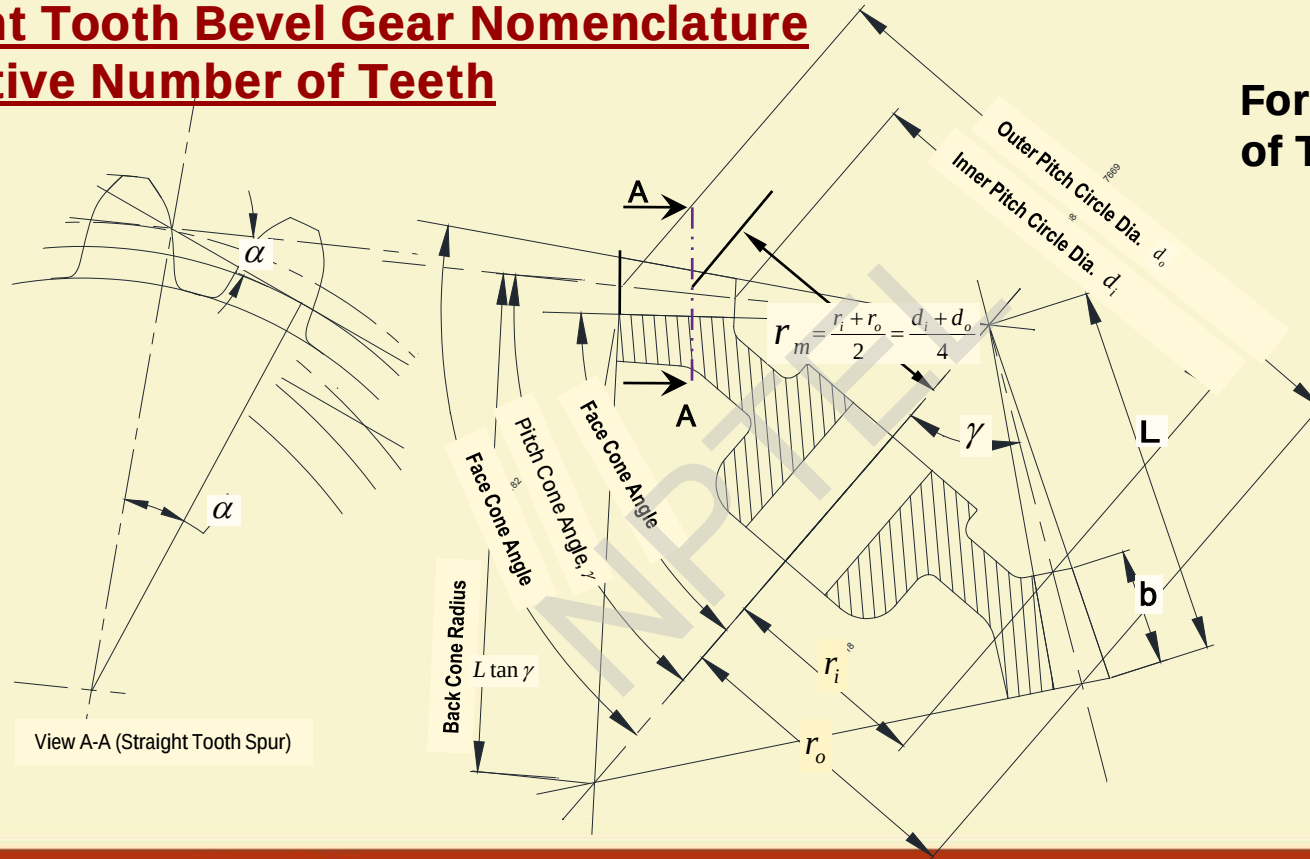


Straight Tooth Bevel Gear Nomenclature

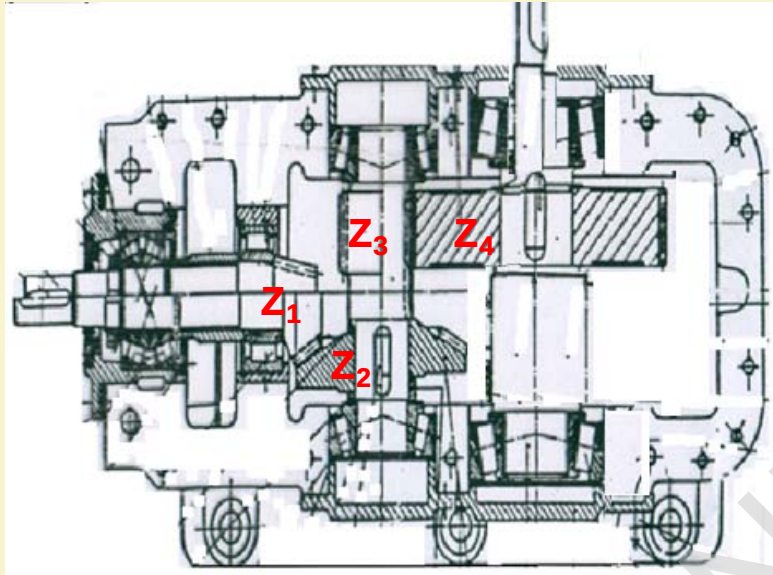
Formative Number of Teeth

Formative Number of Teeth :

$$Z' = \frac{Z}{\cos \gamma}$$



Straight Bevel Gear Design (90° Shafts) (Contd.....) :



Assembled plan view
(Not of the same one as below)



View of 2-stage (Bevel-Helical) gear box.
(Top cover open) : Source- A Leaflet.

Design of a Bevel- Helical Two Stage Gear Box:

Straight Bevel Gear Design (90° Shafts):

Normal Module (m_{bevel} , in meter),

For straight tooth bevel gear, can be estimated as :

$$m_{bevel} = \sqrt[3]{\frac{2T}{\frac{S_o C_v}{C_w} ZY\psi(1-\psi_o)}}$$



Where:

T = Torque (Nm),

Z = Number of teeth,

$$\frac{S_o C_v}{C_w}$$

Can also be replaced by S_d = 'Ultimate strength/3 to 3.5' in first estimation of module. It is applicable for all gears.

C_v = Velocity factor,

= 1 for precision gears & no shock,
1.2 for general purpose gear.

C_w = Wear load / Lubrication factor,

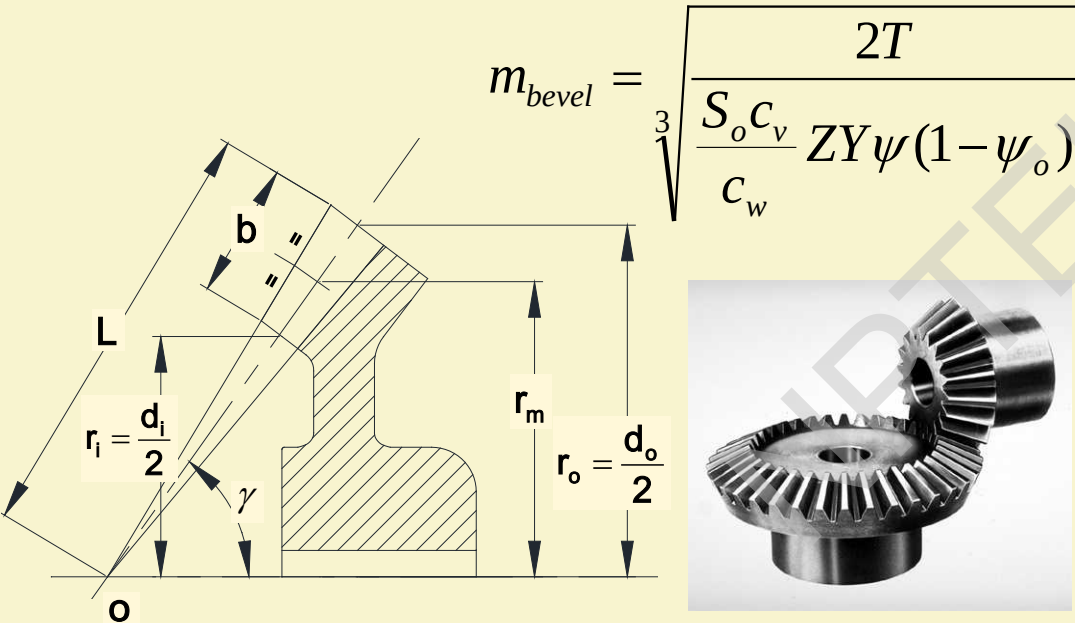
= 1 for force lubrication &
1.5 for sump/splash lubrication

S_o = Allowable design strength (Pas),
= 'Yield Strength/2.5 to 3', for infinite
life of gear.

Straight Tooth Bevel Gear

Straight Bevel Gear Design (90° Shafts):

Normal Module (m_{bevel} , in meter),



Straight Tooth Bevel Gear

ψ = Width factor [active width b of gear/module],
= 10 to 12 or lower for bevel gear.

$\psi_o = b/l$ for bevel gear , usually
1/3 or less,

Modified Lewis form factor

$$Y = \pi(0.154 - 0.912 / Z')$$

and for straight bevel gear ,
Formative number of teeth-

$$Z' = Z / \cos \gamma$$



Straight Bevel Gear Design (90° Shafts):

Normal Module (m_{bevel} , in meter),

$$m_{bevel} = \sqrt[3]{\frac{2T}{\frac{S_o C_v}{C_w} ZY \psi (1 - \psi_o)}}$$

For straight tooth bevel gear:

Mean PCD (Straight Bevel)

$$= 2 \times \text{mean } r = Z \times m_{bevel}$$

Other relations.

$$\gamma_p + \gamma_g = 90^\circ$$

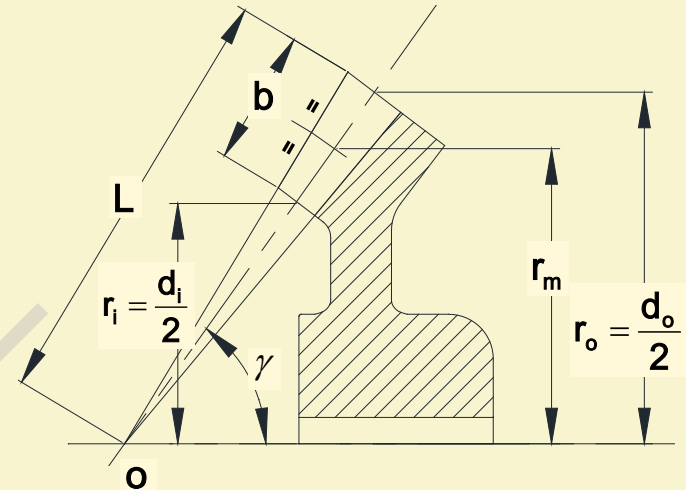
And,

Formative number of teeth for straight bevel gear ,

$$Z' = Z / \cos \gamma$$

γ = pitch cone angle .

$$\tan \gamma_p / \tan \gamma_g = Z_p / Z_g$$



Straight Tooth Bevel Gear

Straight Bevel Gear Design (90° Shafts):

Wear Load Capacity of Straight Tooth Bevel Gear and Probable Dynamic Load:

Wear Load Capacity of Straight Tooth Bevel Gear:

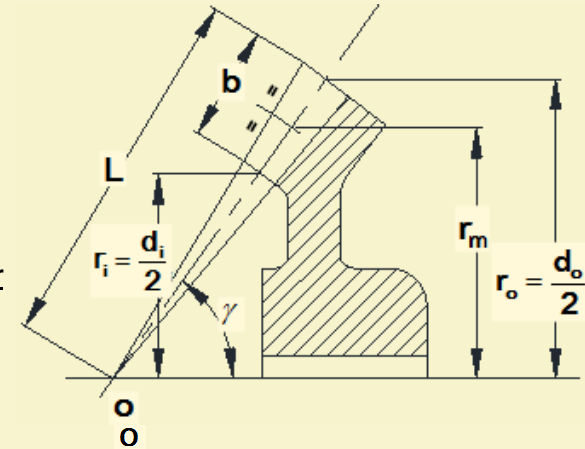
$$F_w = \frac{d_{mp}}{\cos \gamma_p} b K Q_g = \frac{d_{mg}}{\cos \gamma_g} b K Q_p$$

Where,

$$Q_g = \frac{2Z'_g}{Z'_g + Z'_p} \quad \text{and} \quad Q_p = \frac{2Z'_p}{Z'_g + Z'_p}$$



Straight Tooth Bevel Gear



K Is same as defined in case of spur gear against gear and pinion teeth hardness.

Z'_g, Z'_p Formative number of teeth of gear and pinion

And Finally :

$$F_w \geq 1.15 F_d$$

Probable Dynamic Load is Estimated as In Spur Gear

Straight Bevel Gear Design (90° Shafts) (Contd.....) :

Tooth Loads of Straight Tooth Bevel Gear:

Tangential Load $F_t = \frac{T}{r_m}$

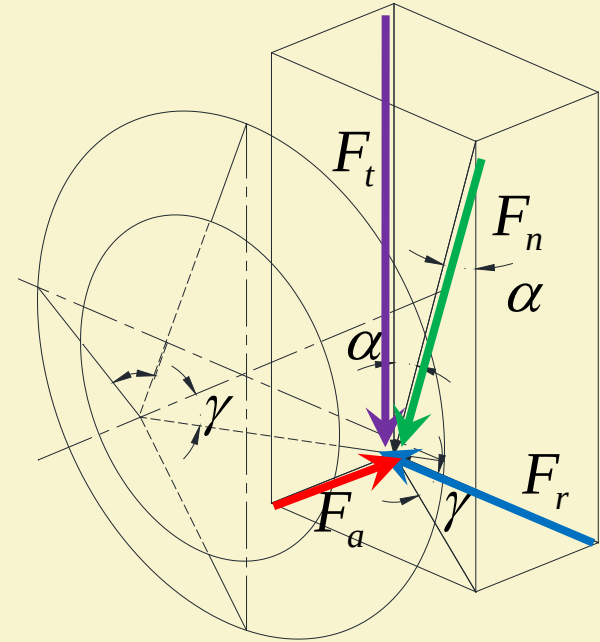
Normal Load $F_n = F_t \sec \alpha$

Radial Load $F_r = F_t \tan \alpha \cos \gamma$

Axial Load $F_a = F_t \tan \alpha \sin \gamma$

It is to be noted that for **90° Shafts** Straight Bevel Gears:

$$F_{tg} = F_{tp} \quad , \quad F_{ag} = F_{rp} \quad \text{and} \quad F_{rg} = F_{ap}$$



Thank You



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Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

Lecture-09

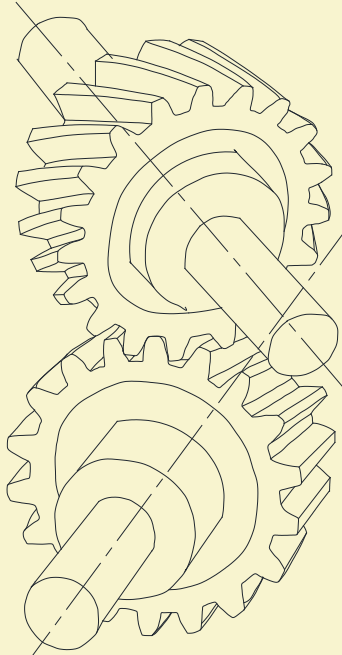
Crossed Helical Gear-I



Outline of the Lecture

- **Introduction to Crossed (Axes) helical Gear,**
 - **Definition & Basic Theory,**
 - **Direction of Helix Angles,**
 - **PCDs and Centre Distance,**
- **Gear Ratio, Speed Ratio & PCD Ratio- Relations,**
 - **Solution for 90° Crossed Helical Gear.**

Crossed Helical Gear

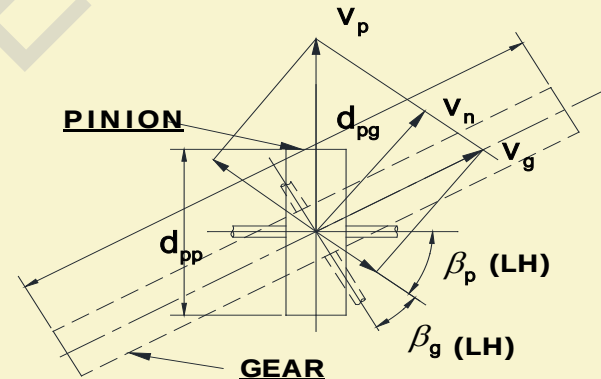


3D View of Helical gears with non parallel shafts (Crossed Helical Gear)

Let a helical pinion having teeth Z_p and helix angle β_p (LH).

Is mating with a helical gear having teeth Z_g and helix angle β_g (LH) as shown in the illustration.

Then the axes of the pinion and the gear are non parallel. It may be called as crossed helical gear also:



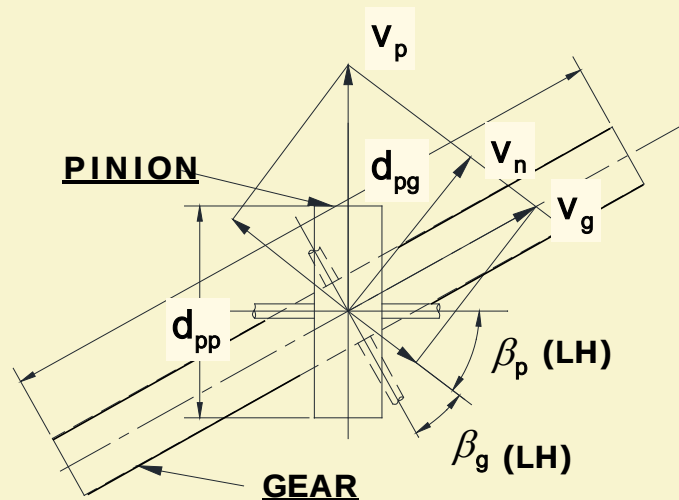
Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :

PCDs of pinion and gear are expressed as:

$$d_{pp} = \frac{Z_p p_{fp}}{\pi} = \frac{Z_p p_n}{\pi \cos \beta_p}$$

$$d_{pg} = \frac{Z_g p_{fg}}{\pi} = \frac{Z_g p_n}{\pi \cos \beta_g}$$



Helical gears with non parallel shafts.

Where, p_{fp} , p_{fg} are face pitches of pinion and gear respectively.
and p_n is the normal circular pitch.

Crossed Helical Gear (contd....) :

As pinion and gear are mating they have common **normal pitch** (and module).

Therefore, the centre distance can be expressed as:

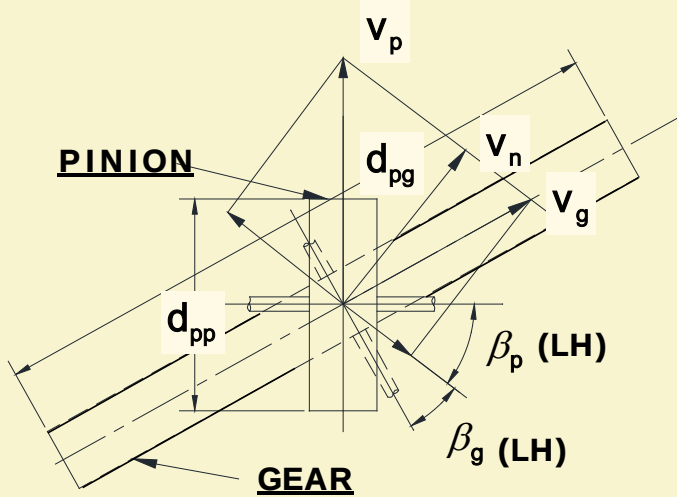
$$A_{CH} = \frac{d_{pp} + d_{pg}}{2} = \frac{p_n}{2\pi} \left[\frac{Z_p}{\cos \beta_p} + \frac{Z_g}{\cos \beta_g} \right]$$

Let the pitch-line velocities of pinion and gear be V_p and V_g respectively. The common velocity V_n normal to the mating teeth surfaces of pinion and gear is expressed as:

$$V_n = V_p \cos \beta_p = V_g \cos \beta_g$$

Or,

$$V_g = \frac{\cos \beta_p}{\cos \beta_g} V_p$$



Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :

The angular velocities pinion and gear are:

$$\omega_p = \frac{2V_p}{d_{pp}} \quad \text{and,} \quad \omega_g = \frac{2V_g}{d_{pg}} = \frac{2 \cos \beta_p}{d_{pg} \cos \beta_g} V_p$$

The velocity ratio ω_p / ω_g is expressed as:

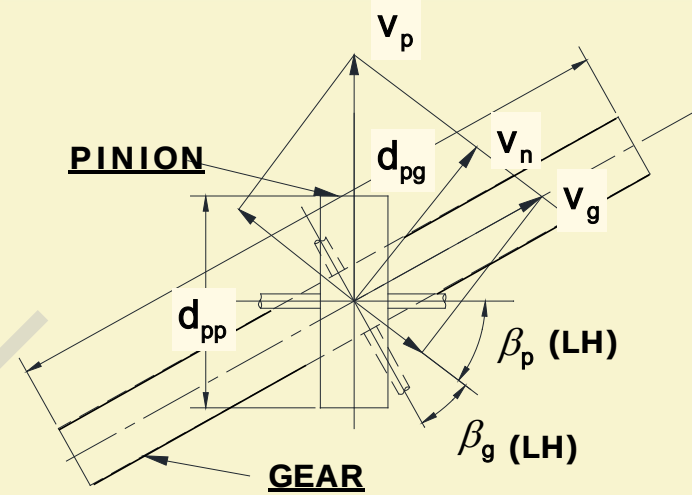
$$\frac{\omega_p}{\omega_g} = \frac{d_{pg} \cos \beta_g}{d_{pp} \cos \beta_p}$$

Substituting the expressions of d_{pp} and d_{pg} :

$$\frac{\omega_p}{\omega_g} = \left[\frac{\frac{Z_g p_n}{\pi \cos \beta_g}}{\frac{Z_p p_n}{\pi \cos \beta_p}} \right] \times \frac{\cos \beta_g}{\cos \beta_p}$$

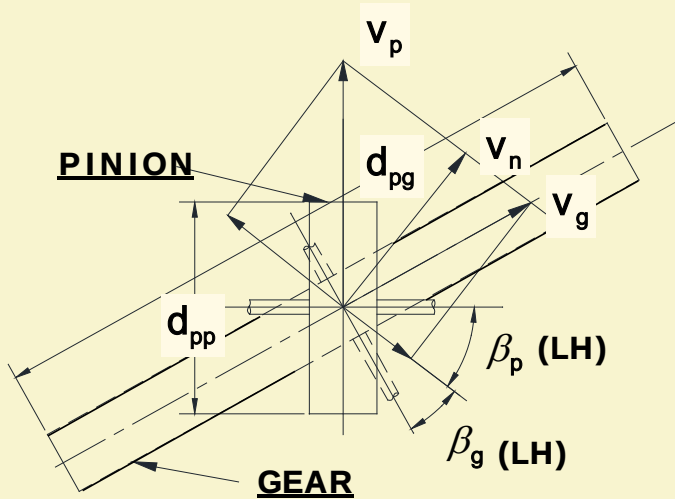
Or,

$$\frac{\omega_p}{\omega_g} = \frac{Z_g}{Z_p}$$



Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :



Helical gears with non parallel shafts.

Thus, the velocity ratio of the gear and pinion,

$$\frac{\omega_p}{\omega_g} = \frac{Z_g}{Z_p}$$

depends on the number of teeth, and independent of helix angles and diameters.

Now, expressing the teeth (Reduction) ratio, as :

$$\frac{Z_g}{Z_p} = \frac{1}{\lambda}$$

Substituting in the expression of the Centre Distance :

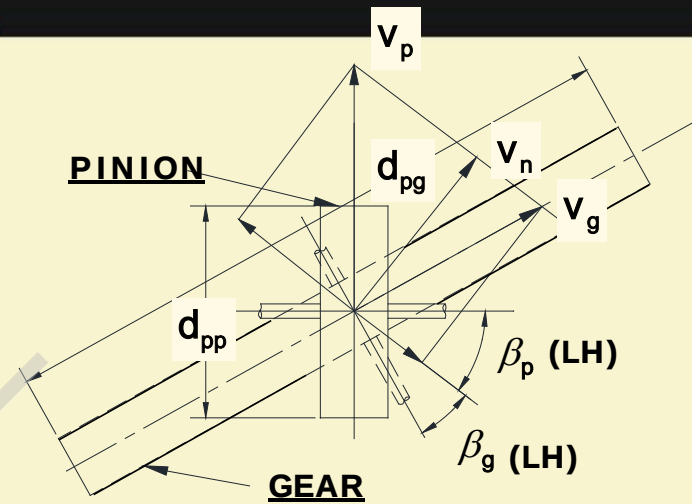
$$\left[\frac{\lambda}{\cos \beta_p} + \frac{1}{\cos \beta_g} \right] = \frac{2 \pi A_{CH}}{Z_g p_n}$$

Crossed Helical Gear (contd....) :

In designing such crossed helical gears or non parallel shaft gears, very often angle between shaft axes i.e., $(\beta_p + \beta_g)$, reduction ratio i.e., $(1/\lambda)$ and A_{CH} are specified.

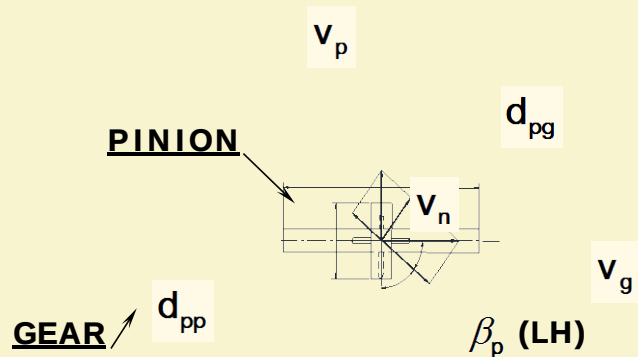
Therefore, Z_g and suitable standard module i.e., p_n (circular pitch) are chosen. Values of β_p and β_g are found out by trial and error to solve:

$$\left[\frac{\lambda}{\cos \beta_p} + \frac{1}{\cos \beta_g} \right] = \frac{2\pi A_{CH}}{Z_g p_n}$$



Helical gears with non parallel shafts.

Solution for 90° Crossed Helical Gear :



$$(\beta_p + \beta_g) = 90^\circ \quad | \quad \beta_g \text{ (LH)}$$

Helical gears with 90° shafts.

Solution for the helical gear and pinion with right angle shaft axes can be obtained using Newton-Raphson numerical method as follows:

$$\text{As, } (\beta_p + \beta_g) = 90^\circ$$

$$\text{Therefore, } \cos \beta_p = \cos (90^\circ - \beta_g) = \sin \beta_g$$

Substituting in the expression of CD:

$$\left[\frac{\lambda}{\sin \beta_g} + \frac{1}{\cos \beta_g} \right] = \frac{2 \pi A_{CH}}{Z_g p_n}$$

Solution for 90° Crossed Helical Gear (contd....) :

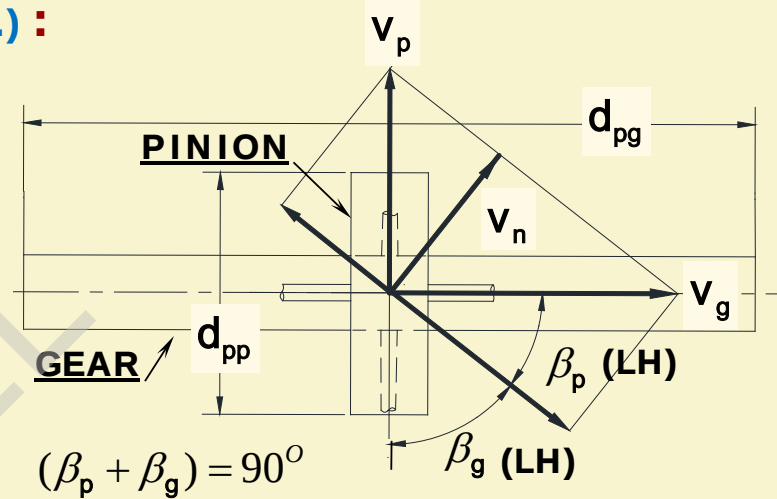
Now, let:
$$\frac{2\pi A_{CH}}{Z_g p_n} = C$$

Therefore, substituting and multiplying both sides by $\sin \beta_g$:

$$\left[\frac{\lambda}{\sin \beta_g} + \frac{1}{\cos \beta_g} \right] \times \sin \beta_g = C \sin \beta_g$$

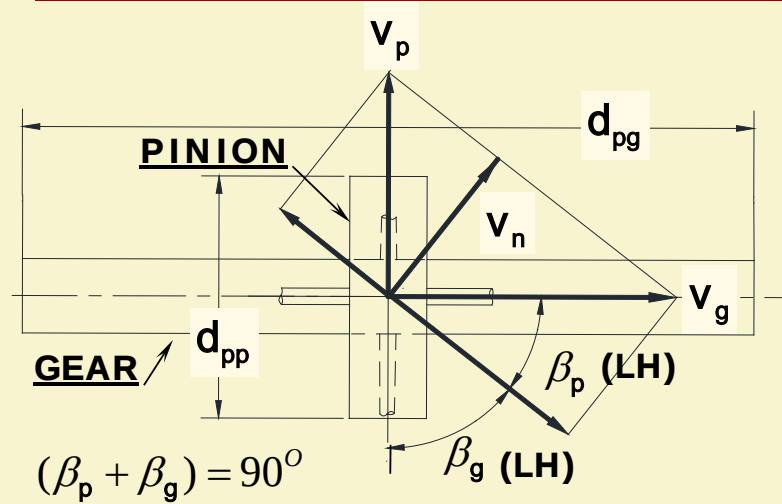
Simplifying and expressing as a function $f(\beta_g)$:

$$f(\beta_g) = \lambda + \tan \beta_g - C \sin \beta_g$$



Helical gears with 90° shafts.

Solution for 90° Crossed Helical Gear :



Helical gears with 90° shafts.

The equation $f(\beta_g) = \lambda + \tan \beta_g - C \sin \beta_g$
in now differentiated with respect to β_g :

$$f'(\beta_g) = \frac{d(f(\beta_g))}{d\beta_g} = \frac{1}{\cos^2 \beta_g} - C \cos \beta_g$$

Now let:
$$h = -\frac{f(\beta_g)}{f'(\beta_g)}$$

Therefore, a closer value of β_g can be chosen as: $\beta'_g = \beta_g + h$

Solution for 90° Crossed Helical Gear (contd....) :

β'_g is now considered as the new trial root in the Newton-Raphson method, i.e. :

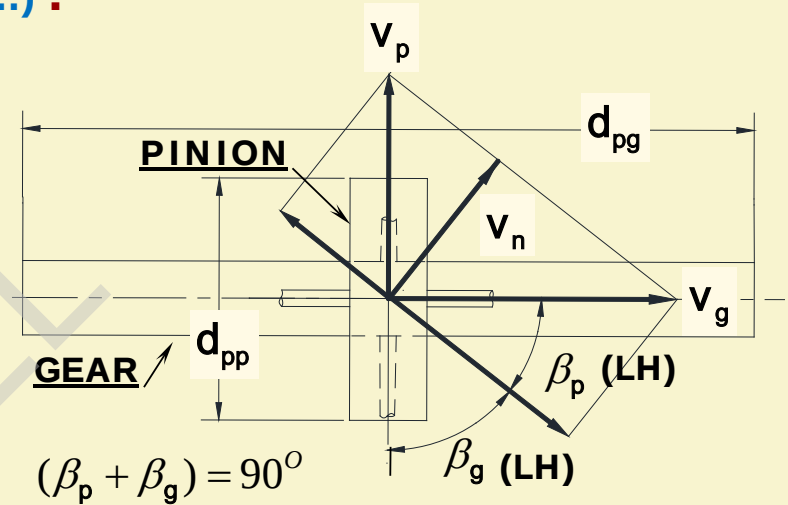
$$f(\beta'_g) = \lambda + \tan \beta'_g - C \sin \beta'_g$$

Differentiating again with respect to β'_g :

$$f'(\beta'_g) = \frac{d(f(\beta'_g))}{d\beta'_g} = \frac{1}{\cos^2 \beta'_g} - C \cos \beta'_g$$

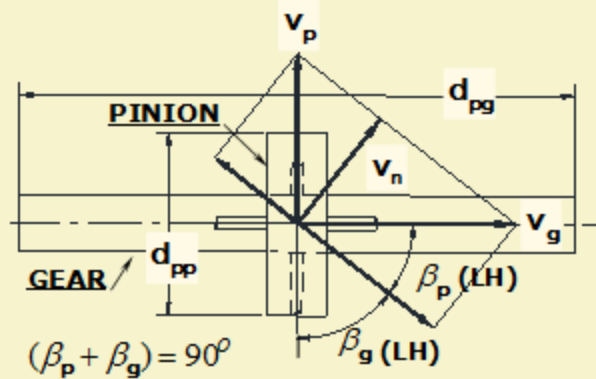
The new incremental value $h' = -\frac{f(\beta'_g)}{f'(\beta'_g)}$ and new root is $\beta''_g = \beta'_g + h'$

The process is repeated till the new root becomes very close to previous value.



Helical gears with 90° shafts.

Solution for 90° Crossed Helical Gear :

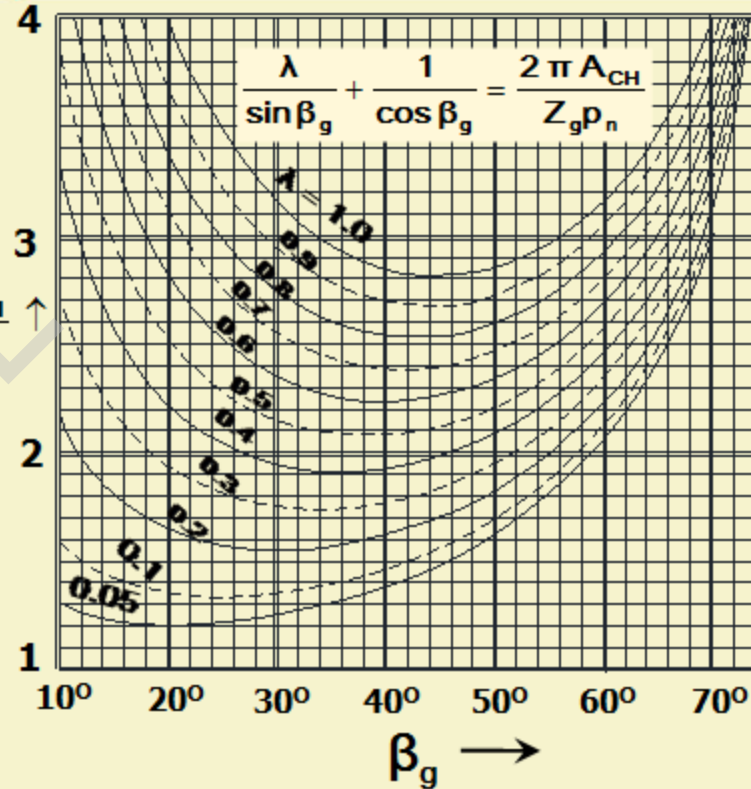


Helical gears with 90° shafts.

The solution can best be revealed through an practical example.

However, the initial trial value of β_g can be assessed from the chart of $C = \frac{2\pi A_{CH}}{Z_g p_n}$ vs β_g for different values of λ .

$$C = \frac{2\pi A_{CH}}{Z_g p_n}$$



Design chart for Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example :

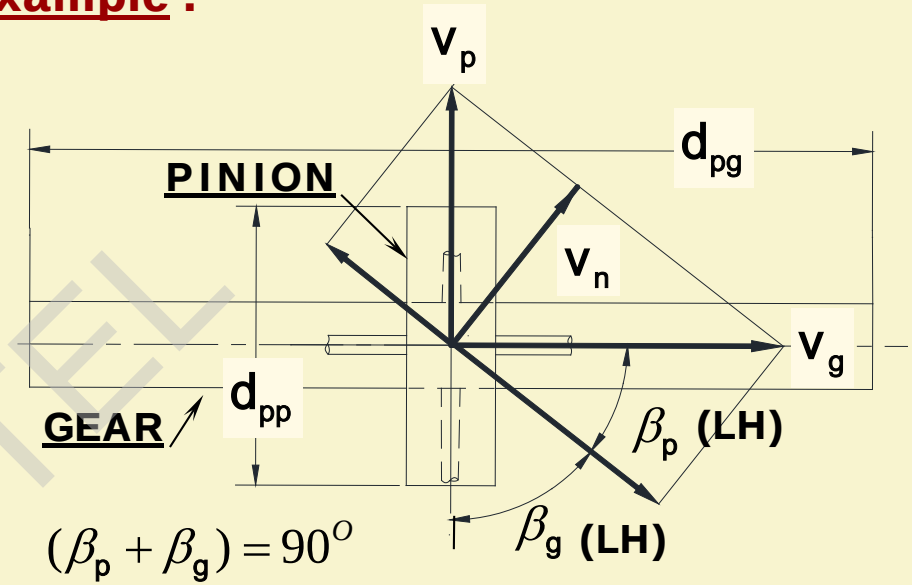
Problem:

A Helical pinion and gear set with 90° shafts have 15 and 58 involute (20°) teeth respectively.

The centre distance is specified as 200 mm with normal module as 5 mm.

Find the appropriate helix angles of the pinion and the gear.

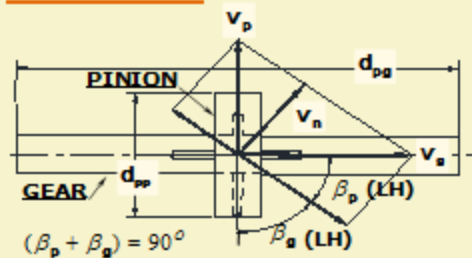
What will be their pitch circle diameters?



Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution:



Helical gears with 90° shafts.

Given:

$$Z_p = 15, \quad Z_g = 58,$$

$$m_n = 5 \text{ mm},$$

$$A_{CH} = 200 \text{ mm},$$

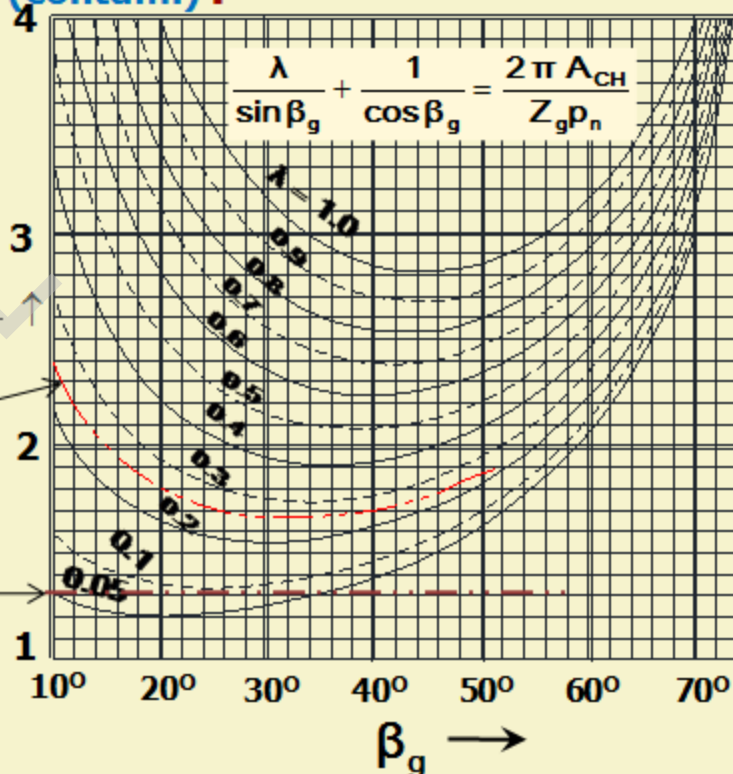
$$\beta_p + \beta_g = 90^\circ$$

To Find: β_p, β_g, d_{pp} & d_{pg} .

$$\lambda = Z_p / Z_g = 15 / 58 = 0.258,$$

$$\text{and } C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 5)} = 1.38$$

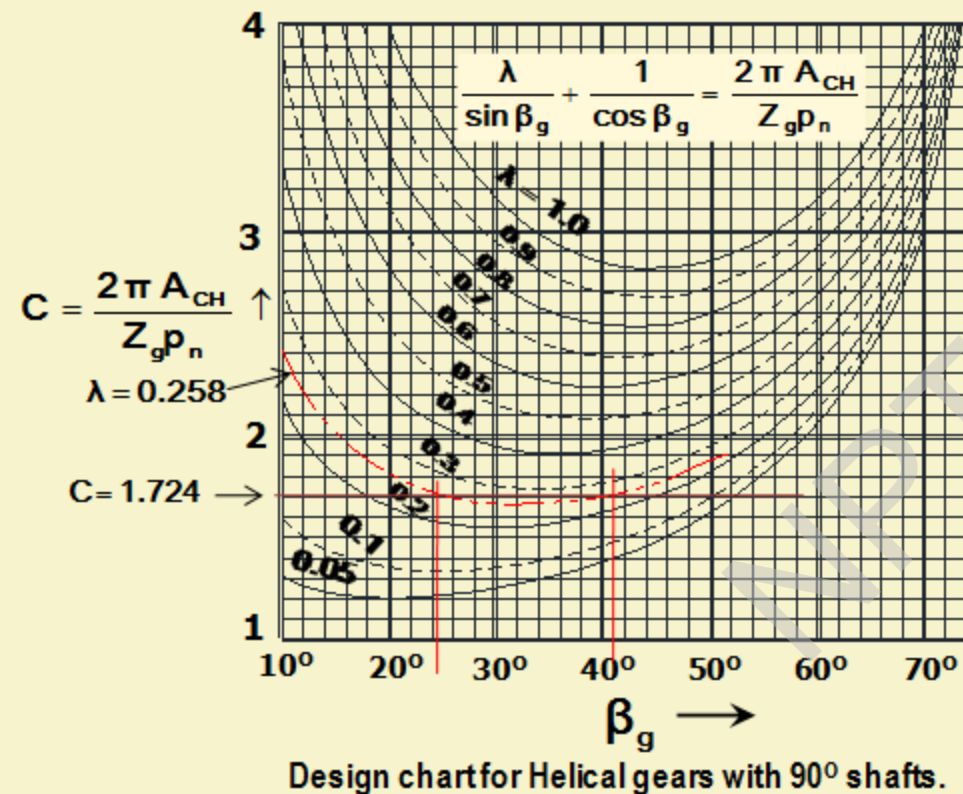
The gear data is not suitable for 90° shafts



Design chart for Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):



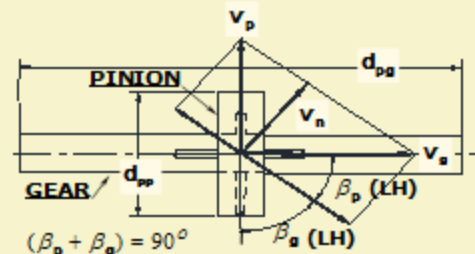
To make the problem feasible the **module** is now taken as **4 mm**.

Therefore, now

$$C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724 ,$$

but $\lambda = Z_p / Z_g = 0.258$, remains unchanged.

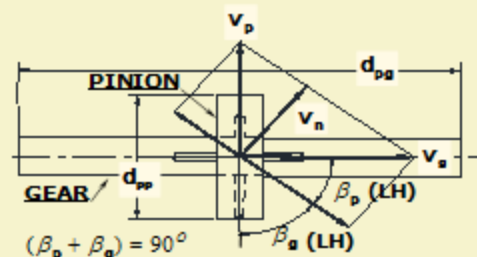
From the graph the initial value of β_g may be taken as **24.5° or 41°**.



Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....):

Solution (Contd...):



Helical gears with 90° shafts.

Finally:

$$Z_p = 15, \quad Z_g = 58, \quad A_{CH} = 200 \text{ mm}, \quad m_n = 4 \text{ mm, instead of 5mm.}$$

$$\beta_p + \beta_g = 90^\circ$$

To Find: β_p, β_g, d_{pp} & d_{pg} .

$$C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

and $\lambda = Z_p / Z_g = 15 / 58 = 0.258,$

From the graph the initial value of

β_g may be taken as 24.5° or 41° .

Thank You



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Prof. Rathindranath Maiti
Mechanical Engineering Department

Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

Lecture-09

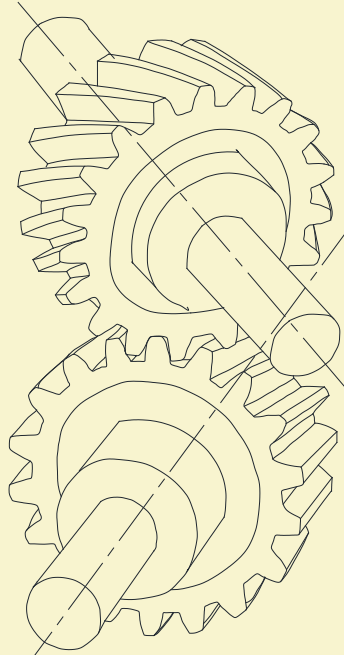
Crossed Helical Gear-I



Outline of the Lecture

- **Introduction to Crossed (Axes) helical Gear,**
 - **Definition & Basic Theory,**
 - **Direction of Helix Angles,**
 - **PCDs and Centre Distance,**
- **Gear Ratio, Speed Ratio & PCD Ratio- Relations,**
 - **Solution for 90° Crossed Helical Gear.**

Crossed Helical Gear

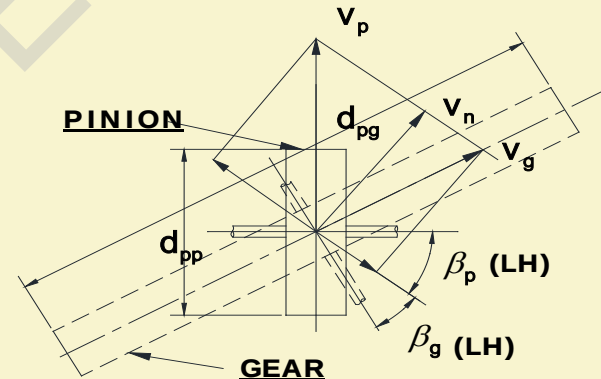


3D View of Helical gears with non parallel shafts (Crossed Helical Gear)

Let a helical pinion having teeth Z_p and helix angle β_p (LH).

Is mating with a helical gear having teeth Z_g and helix angle β_g (LH) as shown in the illustration.

Then the axes of the pinion and the gear are non parallel. It may be called as crossed helical gear also:



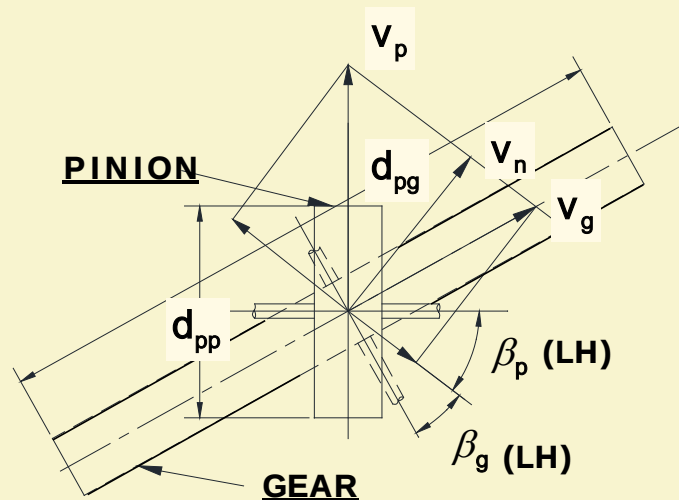
Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :

PCDs of pinion and gear are expressed as:

$$d_{pp} = \frac{Z_p p_{fp}}{\pi} = \frac{Z_p p_n}{\pi \cos \beta_p}$$

$$d_{pg} = \frac{Z_g p_{fg}}{\pi} = \frac{Z_g p_n}{\pi \cos \beta_g}$$



Helical gears with non parallel shafts.

Where, p_{fp} , p_{fg} are face pitches of pinion and gear respectively.
and p_n is the normal circular pitch.

Crossed Helical Gear (contd....) :

As pinion and gear are mating they have common **normal pitch** (and module).

Therefore, the centre distance can be expressed as:

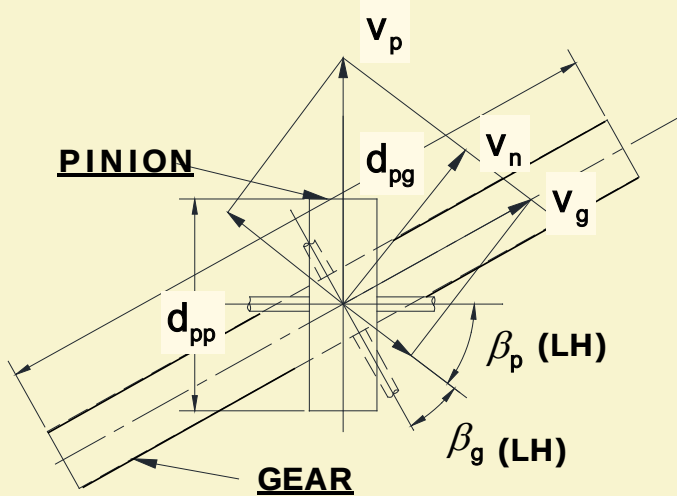
$$A_{CH} = \frac{d_{pp} + d_{pg}}{2} = \frac{p_n}{2\pi} \left[\frac{Z_p}{\cos \beta_p} + \frac{Z_g}{\cos \beta_g} \right]$$

Let the pitch-line velocities of pinion and gear be V_p and V_g respectively. The common velocity V_n normal to the mating teeth surfaces of pinion and gear is expressed as:

$$V_n = V_p \cos \beta_p = V_g \cos \beta_g$$

Or,

$$V_g = \frac{\cos \beta_p}{\cos \beta_g} V_p$$



Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :

The angular velocities pinion and gear are:

$$\omega_p = \frac{2V_p}{d_{pp}} \quad \text{and,} \quad \omega_g = \frac{2V_g}{d_{pg}} = \frac{2 \cos \beta_p}{d_{pg} \cos \beta_g} V_p$$

The velocity ratio ω_p / ω_g is expressed as:

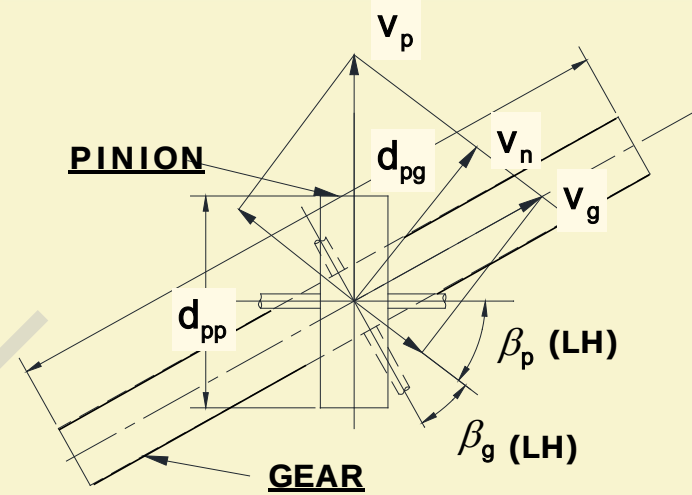
$$\frac{\omega_p}{\omega_g} = \frac{d_{pg} \cos \beta_g}{d_{pp} \cos \beta_p}$$

Substituting the expressions of d_{pp} and d_{pg} :

$$\frac{\omega_p}{\omega_g} = \left[\frac{\frac{Z_g p_n}{\pi \cos \beta_g}}{\frac{Z_p p_n}{\pi \cos \beta_p}} \right] \times \frac{\cos \beta_g}{\cos \beta_p}$$

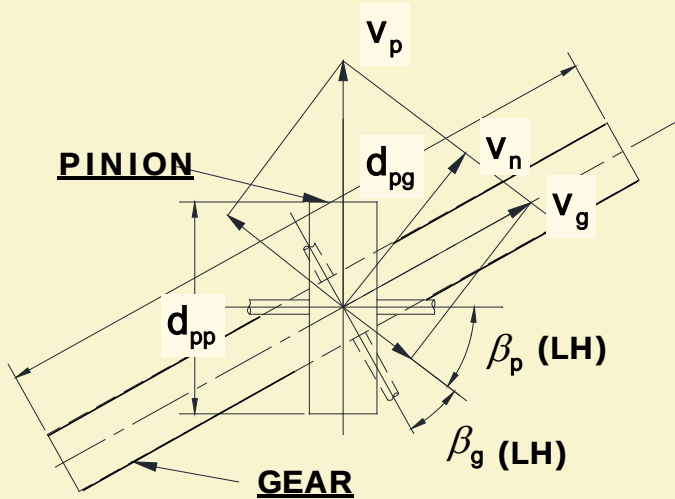
Or,

$$\frac{\omega_p}{\omega_g} = \frac{Z_g}{Z_p}$$



Helical gears with non parallel shafts.

Crossed Helical Gear (contd....) :



Helical gears with non parallel shafts.

Thus, the velocity ratio of the gear and pinion,

$$\frac{\omega_p}{\omega_g} = \frac{Z_g}{Z_p}$$

depends on the number of teeth, and independent of helix angles and diameters.

Now, expressing the teeth (Reduction) ratio, as :

$$\frac{Z_g}{Z_p} = \frac{1}{\lambda}$$

Substituting in the expression of the Centre Distance :

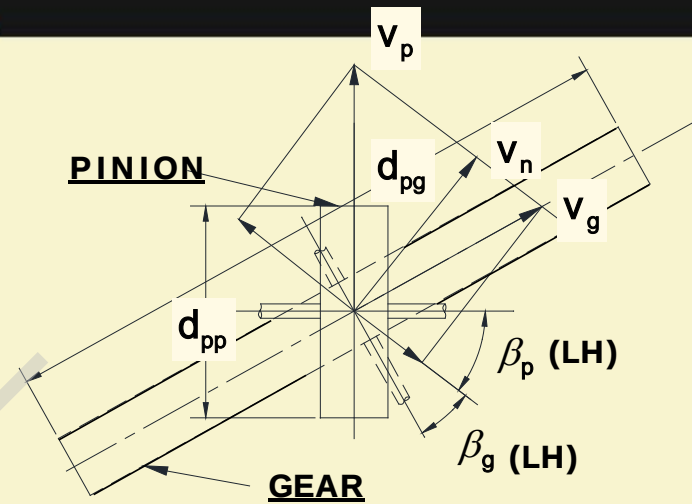
$$\left[\frac{\lambda}{\cos \beta_p} + \frac{1}{\cos \beta_g} \right] = \frac{2 \pi A_{CH}}{Z_g p_n}$$

Crossed Helical Gear (contd....) :

In designing such crossed helical gears or non parallel shaft gears, very often angle between shaft axes i.e., $(\beta_p + \beta_g)$, reduction ratio i.e., $(1/\lambda)$ and A_{CH} are specified.

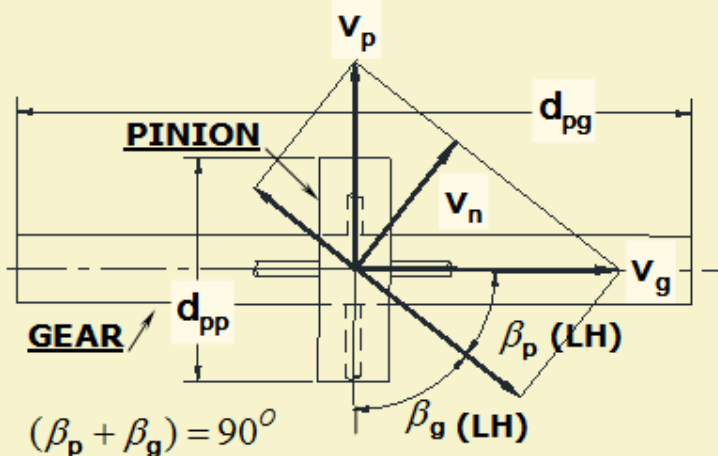
Therefore, Z_g and suitable standard module i.e., p_n (circular pitch) are chosen. Values of β_p and β_g are found out by trial and error to solve:

$$\left[\frac{\lambda}{\cos \beta_p} + \frac{1}{\cos \beta_g} \right] = \frac{2\pi A_{CH}}{Z_g p_n}$$



Helical gears with non parallel shafts.

Solution for 90° Crossed Helical Gear :



Helical gears with 90° shafts.

Solution for the helical gear and pinion with right angle shaft axes can be obtained using Newton-Raphson numerical method as follows:

As, $(\beta_p + \beta_g) = 90^\circ$

Therefore, $\cos \beta_p = \cos (90^\circ - \beta_g) = \sin \beta_g$

Substituting in the expression of CD:

$$\left[\frac{\lambda}{\sin \beta_g} + \frac{1}{\cos \beta_g} \right] = \frac{2\pi A_{CH}}{Z_g p_n}$$

Solution for 90° Crossed Helical Gear (contd....) :

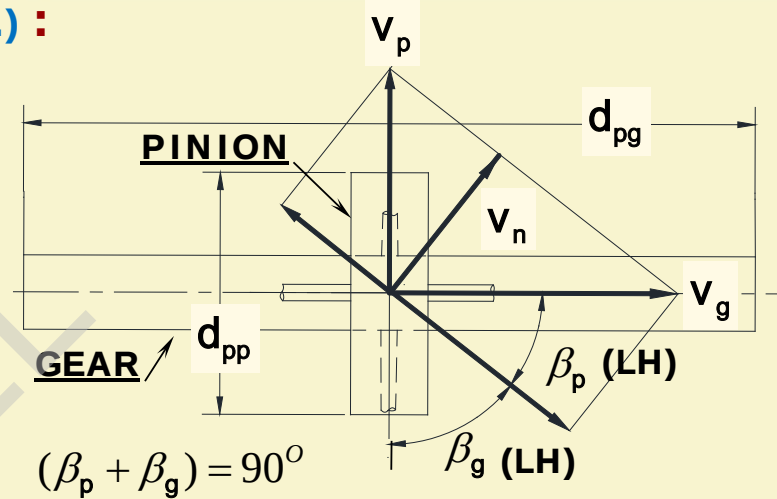
Now, let:
$$\frac{2\pi A_{CH}}{Z_g p_n} = C$$

Therefore, substituting and multiplying both sides by $\sin \beta_g$:

$$\left[\frac{\lambda}{\sin \beta_g} + \frac{1}{\cos \beta_g} \right] \times \sin \beta_g = C \sin \beta_g$$

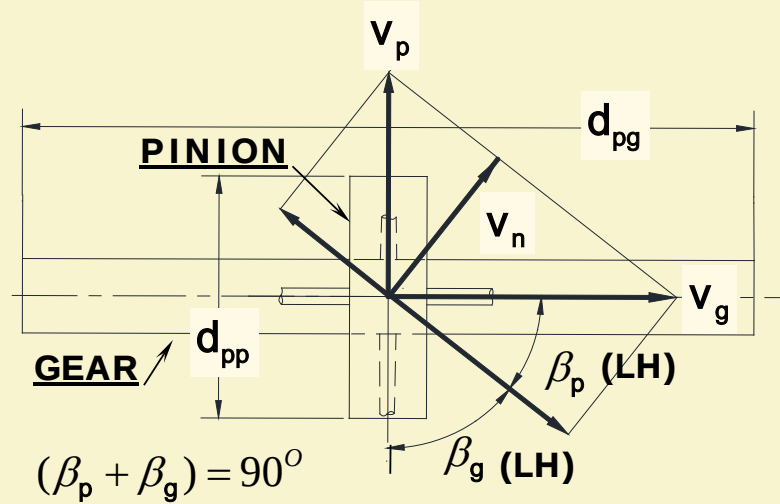
Simplifying and expressing as a function $f(\beta_g)$:

$$f(\beta_g) = \lambda + \tan \beta_g - C \sin \beta_g$$



Helical gears with 90° shafts.

Solution for 90° Crossed Helical Gear :



Helical gears with 90° shafts.

The equation $f(\beta_g) = \lambda + \tan \beta_g - C \sin \beta_g$
in now differentiated with respect to β_g :

$$f'(\beta_g) = \frac{d(f(\beta_g))}{d\beta_g} = \frac{1}{\cos^2 \beta_g} - C \cos \beta_g$$

Now let:
$$h = -\frac{f(\beta_g)}{f'(\beta_g)}$$

Therefore, a closer value of β_g can be chosen as: $\beta'_g = \beta_g + h$

Solution for 90° Crossed Helical Gear (contd....) :

β'_g is now considered as the new trial root in the Newton-Raphson method, i.e. :

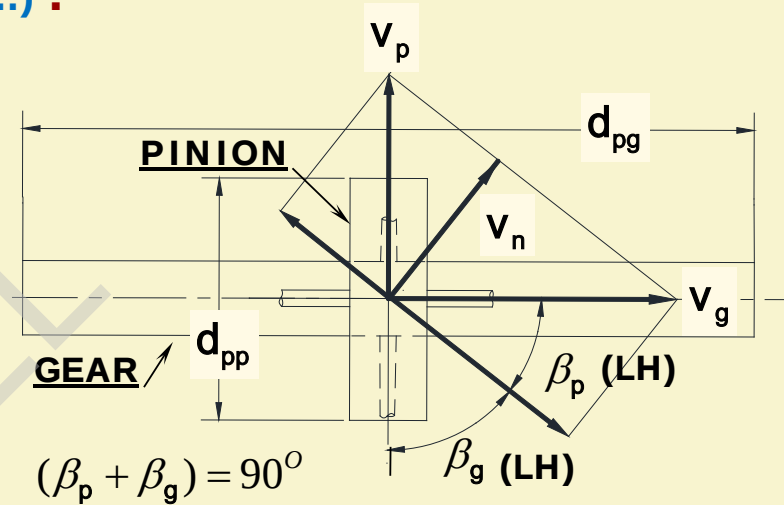
$$f(\beta'_g) = \lambda + \tan \beta'_g - C \sin \beta'_g$$

Differentiating again with respect to β'_g :

$$f'(\beta'_g) = \frac{d(f(\beta'_g))}{d\beta'_g} = \frac{1}{\cos^2 \beta'_g} - C \cos \beta'_g$$

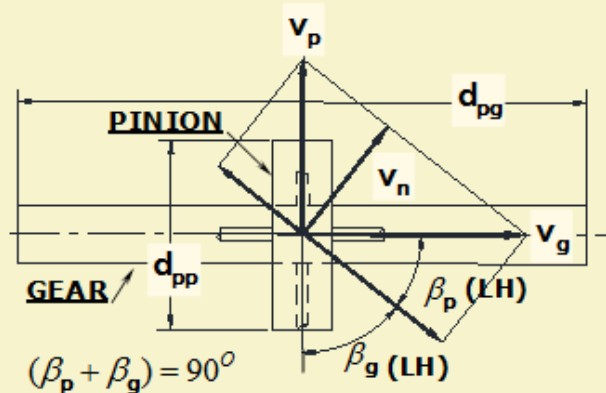
The new incremental value $h' = -\frac{f(\beta'_g)}{f'(\beta'_g)}$ and new root is $\beta''_g = \beta'_g + h'$

The process is repeated till the new root becomes very close to previous value.



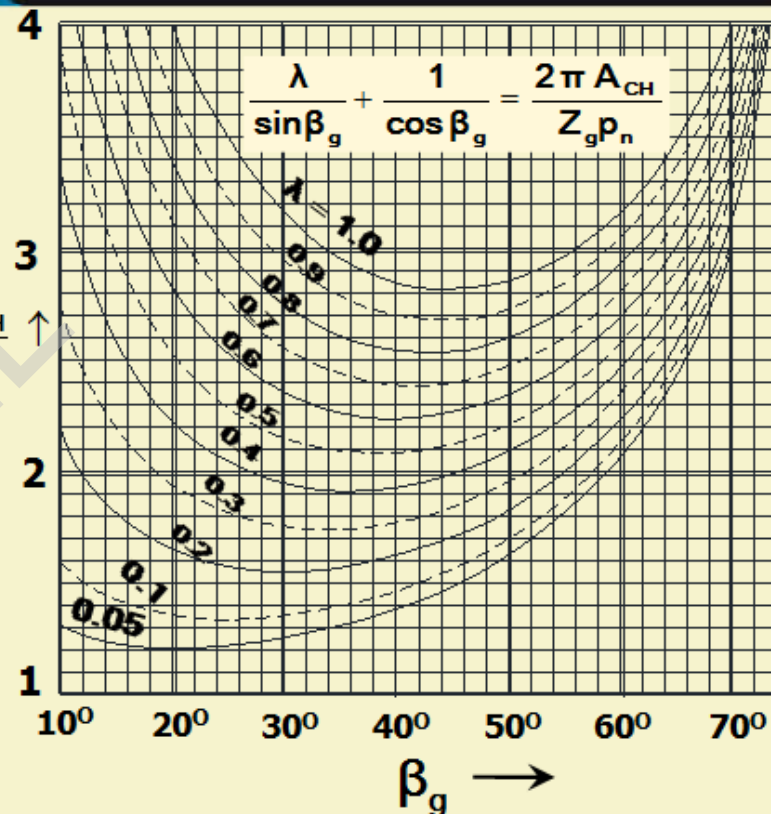
Helical gears with 90° shafts.

Solution for 90° Crossed Helical Gear :



Helical gears with 90° shafts.

$$C = \frac{2\pi A_{CH}}{Z_g p_n}$$



Design chart for Helical gears with 90° shafts.

The solution can best be revealed through an practical example.

However, the initial trial value of β_g can be assessed from the chart of $C = \frac{2\pi A_{CH}}{Z_g p_n}$ vs β_g for different values of λ .

90° Crossed Helical Gear- A Practical Example :

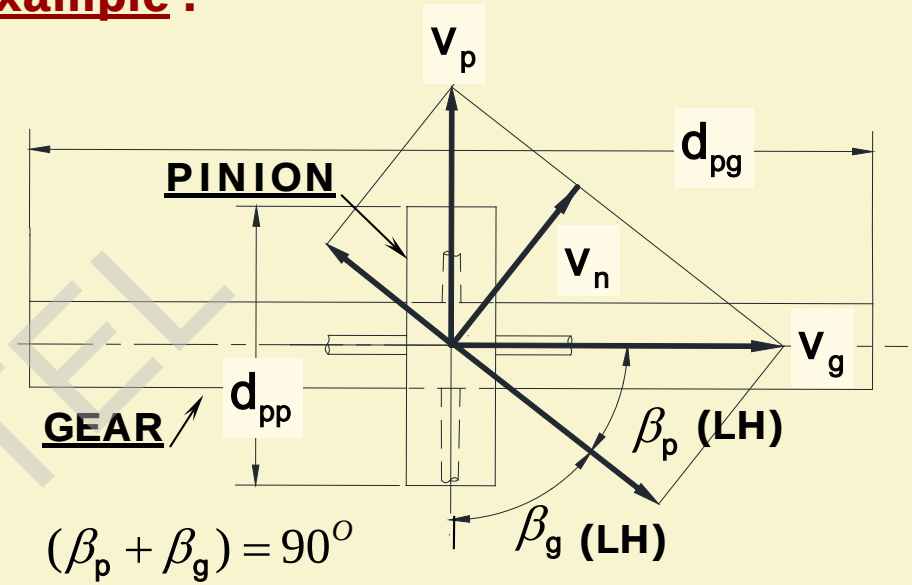
Problem:

A Helical pinion and gear set with 90° shafts have 15 and 58 involute (20°) teeth respectively.

The centre distance is specified as 200 mm with normal module as 5 mm.

Find the appropriate helix angles of the pinion and the gear.

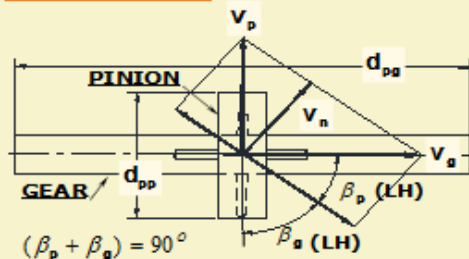
What will be their pitch circle diameters?



Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....):

Solution:



Helical gears with 90° shafts.

Given:

$$Z_p = 15, \quad Z_g = 58,$$

$$m_n = 5 \text{ mm},$$

$$A_{CH} = 200 \text{ mm},$$

$$\beta_p + \beta_g = 90^\circ$$

To Find: β_p , β_g , d_{pp} & d_{pg} .

$$\lambda = Z_p / Z_g = 15 / 58 = 0.258,$$

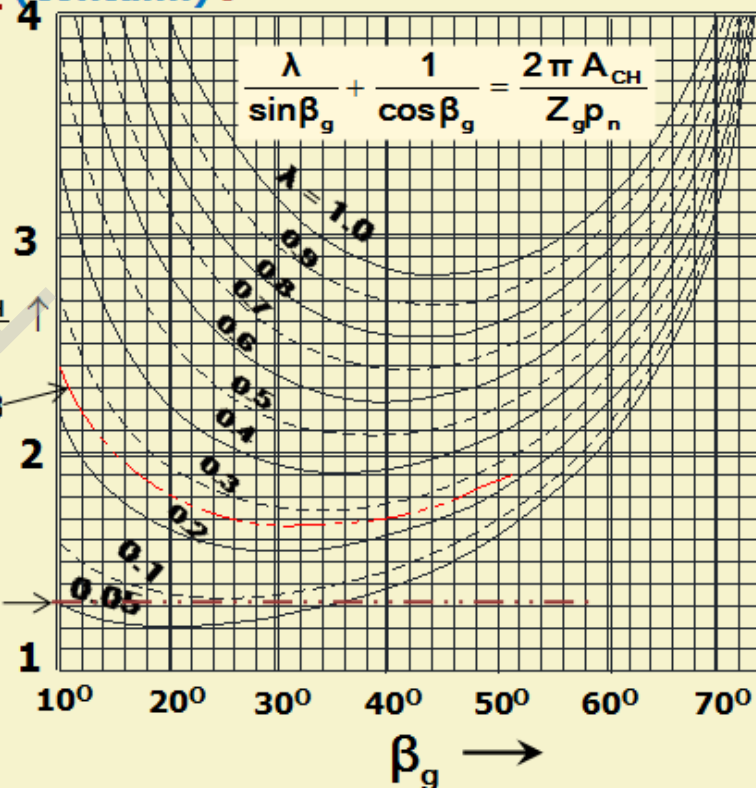
$$\text{and } C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 5)} = 1.38$$

The gear data is
not suitable for
90° shafts

$$C = \frac{2 \pi A_{CH}}{Z_g p_n}$$

$$\lambda = 0.258$$

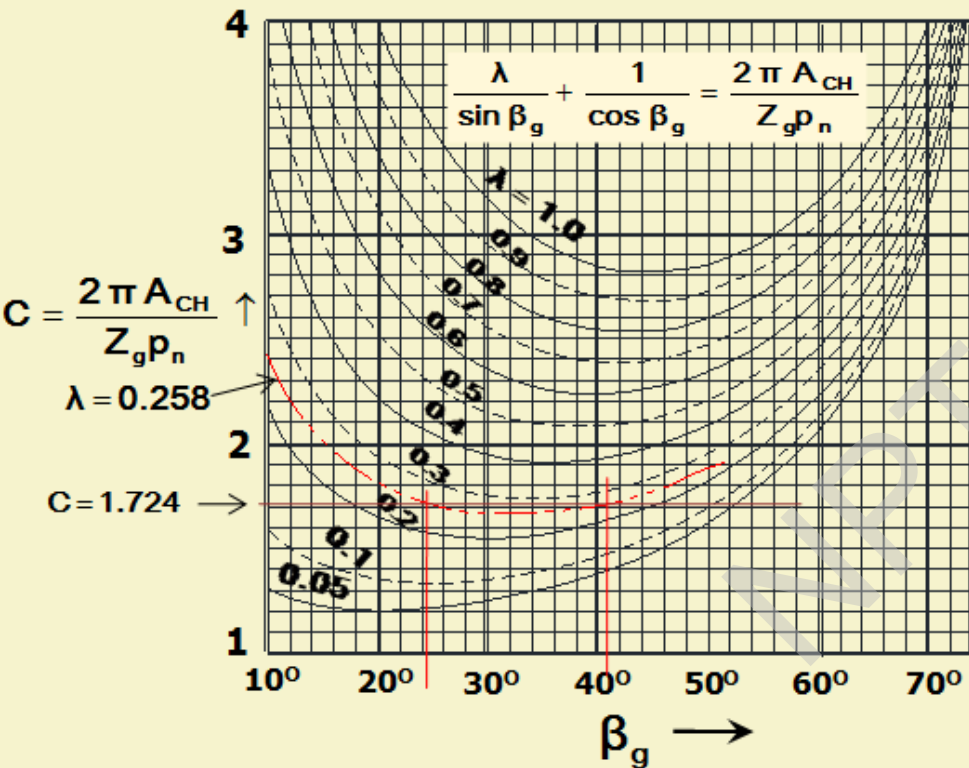
$$C = 1.38$$



Design chart for Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):



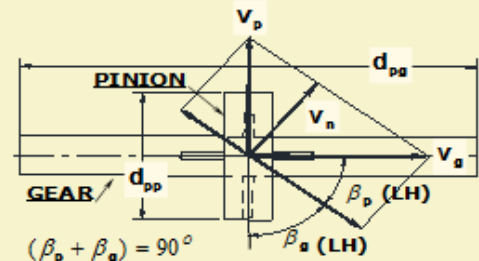
To make the problem feasible the **module** is now taken as **4 mm**.

Therefore, now

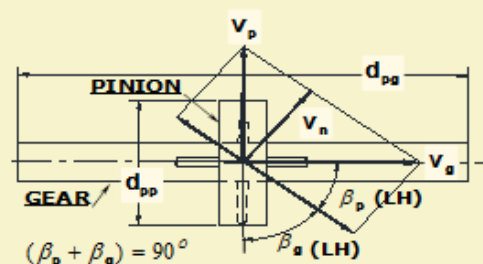
$$C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724 ,$$

but $\lambda = Z_p / Z_g = 0.258$, remains unchanged.

From the graph the initial value of β_g may be taken as **24.5° or 41°**.



Helical gears with 90° shafts.



Helical gears with 90° shafts.

Finally:

$$Z_p = 15, \quad Z_g = 58, \quad A_{CH} = 200 \text{ mm}, \quad m_n = 4 \text{ mm, instead of 5mm.}$$

$$\beta_p + \beta_g = 90^\circ$$

To Find: $\beta_p, \beta_g, d_{pp} \text{ \& } d_{pg}.$

$$C = \frac{2 \pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

and $\lambda = Z_p / Z_g = 15 / 58 = 0.258,$

From the graph the initial value of β_g may be taken as 24.5° or 41°.

Thank You



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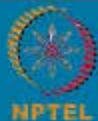
Prof. Rathindranath Maiti
Mechanical Engineering Department

Module-02

Design of Spur (Straight and Helical), Bevel and Worm gears

Lecture-10

Crossed Helical Gear-II and Worm Gear

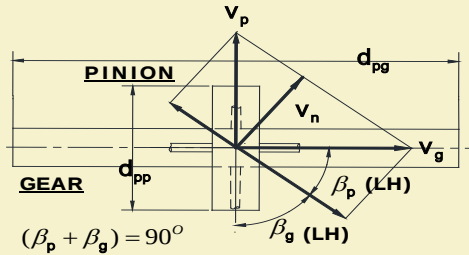


Outline of the Lecture

- **A Workout Example of 90° Crossed Axes Helical Gear (Continued from earlier Lecture),**
- **Choice of Initial Helix Angle of Gear- Limitations,**
- **Transformation of 90° Crossed Axes Helical Gear to Worm Gear,**
- **Similarities and differences in 90° Crossed Axes Helical Gear and Worm Gear,**
- **Strength Basis Design of Crossed Axes Helical Gear and Worm Gear.**

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):



Final Given Data:

$$\begin{aligned} Z_p &= 15, & Z_g &= 58, \\ A_{CH} &= 200 \text{ mm}, \\ m_n &= 4 \text{ mm}, \\ \beta_p + \beta_g &= 90^\circ \end{aligned}$$

Helical gears with 90° shafts.

To Find: β_p, β_g, d_{pp} & d_{pg} .

$$\lambda = Z_p / Z_g = 15 / 58 = 0.258,$$

$$\text{and } C = \frac{2\pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

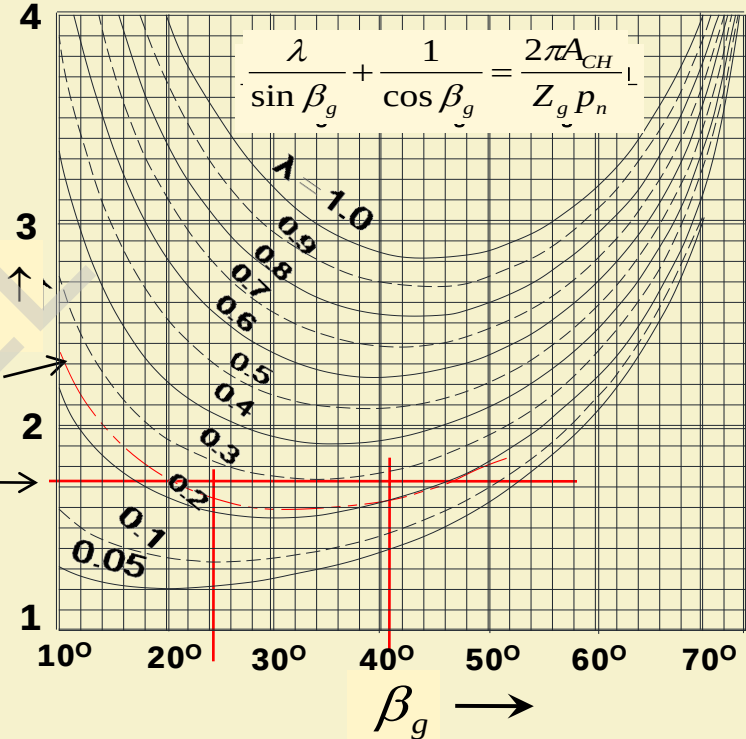
From the graph the initial value of

β_g may be taken as 24.5° or 41°.

$$C = \frac{2\pi A_{CH}}{Z_g p_n}$$

$$\lambda = 0.258$$

$$C = 1.724$$



Design chart for Helical gears with 90° shafts.

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):

Let $\beta_g = 24.5^\circ$, then function $f(\beta_g)$,
following Newton-Raphson method is:

$$f(\beta_g) = \lambda + \tan \beta_g - C \sin \beta_g = 0.258 + \tan(24.5^\circ) - 1.724 \times \sin(24.5^\circ)$$

$$\text{Or, } f(\beta_g) = 0.258 + 0.456 - 0.715 = -0.001$$

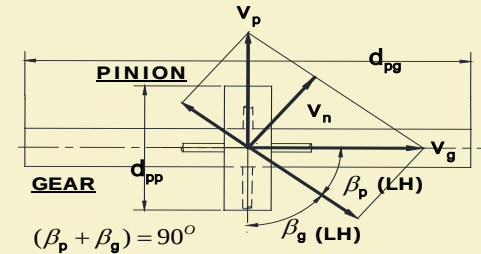
$$\text{and } f'(\beta_g) = \frac{1}{\cos^2 \beta_g} - C \cos \beta_g = \frac{1}{\cos^2(24.5^\circ)} - C \cos(24.5^\circ)$$

$$\text{Or, } f'(\beta_g) = \frac{1}{(0.90996)^2} - 1.724 \times 0.90996 = 1.2077 - 1.56877 = -0.3611$$

$$\text{Therefore, } h = -\frac{f(\beta_g)}{f'(\beta_g)} = -\frac{-0.001}{-0.3611} = -0.00277 \text{ rad} = -0.1587^\circ$$

Now a closer value of β_g can be chosen as:

$$\begin{aligned}\beta'_g &= \beta_g + h \\ &= 24.5^\circ - 0.1587^\circ = 24.3413^\circ\end{aligned}$$



Helical gears with 90° shafts.

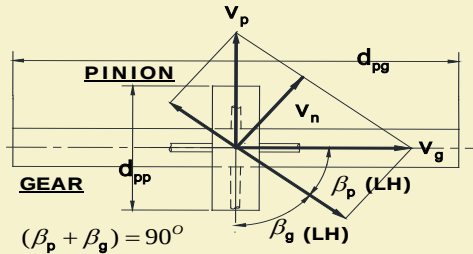
$$C = \frac{2\pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

$$\lambda = Z_p / Z_g = 15/58 = 0.258,$$

From the graph the initial value of β_g may be taken as 24.5° or 41° .

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):



Helical gears with 90° shafts.

$$C = \frac{2\pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

$$\lambda = Z_p / Z_g = 15/58 = 0.258,$$

From the graph the initial value of β_g may be taken as 24.5° or 41°.

Following Newton-Raphson method: $f'(\beta_g)$

$$f(\beta'_g) = \lambda + \tan \beta'_g - C \sin \beta'_g = 0.258 + \tan(24.3413^\circ) - 1.724 \times \sin(24.3413^\circ)$$

$$\text{Or, } f(\beta'_g) = 0.258 + 0.4524 - 0.7106 = -0.0002$$

$$\text{and } f'(\beta'_g) = \frac{1}{\cos^2 \beta'_g} - C \cos \beta'_g = \frac{1}{\cos^2(24.3413^\circ)} - C \cos(24.3413^\circ)$$

$$\text{Or, } f'(\beta'_g) = \frac{1}{(0.9111)^2} - 1.724 \times 0.9111 = 1.2046 - 1.571 = -0.36614$$

$$\text{Therefore, } h' = -\frac{f(\beta'_g)}{f'(\beta'_g)} = -\frac{-0.0002}{-0.36614} = -0.000547 \text{ rad} = -0.0313^\circ.$$

The increment is small.

However, β_g is further refined to :

$$\beta_g = \beta'_g + h' = 24.3413^\circ - 0.0313^\circ = 24.313^\circ$$

$$\text{or, } \beta_g = 24^\circ 18' 47''$$

90° Crossed Helical Gear- A Practical Example (contd....) :

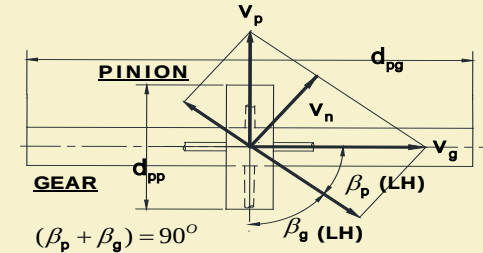
Solution (Contd...):

Now $\beta_g + \beta_p = 90^\circ$
 $\therefore \beta_p = 90^\circ - 24^\circ 18' 47'' = 65^\circ 41' 13''$

This solution is acceptable. The direction of helix in both cases will either be RH or LH.

Similarly, with an initial value of $\beta_g = 41^\circ$ another set of solution is possible.

It is to be noted that the first set of solution is close to worm gearing where as, the second set will remain as helical gearing.



Helical gears with 90° shafts.

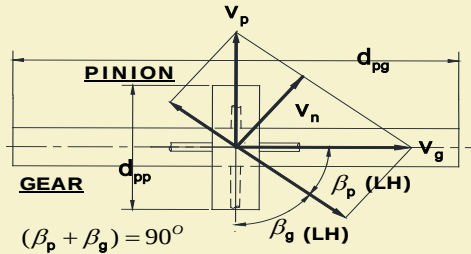
$$C = \frac{2\pi A_{CH}}{Z_g p_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$

$$\lambda = Z_p / Z_g = 15/58 = 0.258,$$

From the graph the initial value of β_g may be taken as 24.5° or 41° .

90° Crossed Helical Gear- A Practical Example (contd....) :

Solution (Contd...):



Helical gears with 90° shafts.

$$C = \frac{2\pi A_{CH}}{Z_g P_n} = \frac{2 \times \pi \times 200}{58 \times (\pi \times 4)} = 1.724$$
$$\therefore Z_p / Z_g = 15 / 58 = 0.258,$$

From the graph the initial value of β_g may be taken as 24.5° or 41°.

The dimensions of the pinion-gear set with the first (final) set of solution are as follows:

$$d_{pp} = \frac{Z_p m_n}{\cos \beta_p} = \frac{15 \times 4}{\cos(65.687^\circ)} = \frac{60}{0.4117} = 145.73 \text{ mm}$$

$$d_{pg} = \frac{Z_g m_n}{\cos \beta_g} = \frac{58 \times 4}{\cos(24.313^\circ)} = \frac{232}{0.9113} = 254.58 \text{ mm}$$

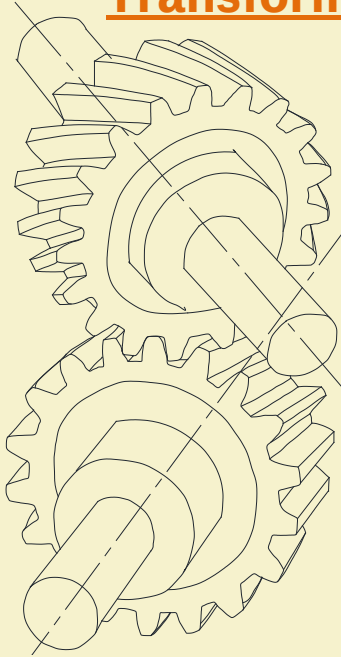
$$A_{CH} = (d_{pp} + d_{pg}) / 2 = (145.73 + 254.58) / 2 = 200.155 \text{ mm}$$

The centre distance is in higher side. Therefore, Newton-Raphson method may be continued further to refine β_g . However, a little over cut of the gear only or both pinion and gear will allow to make the CD = 200 mm.

For the strength basis design of such gear same procedure as in helical gear can be followed. However, pitch line velocity and thereby C_v factors will be different.

Worm Gear Design (Worm and Worm Wheel) :

Transformation from 90° from Crossed Helical Gear



3D View of Helical gears with non parallel shafts (Crossed Helical Gear)

The Worm Gearing may be consider as a special case of 90 degree crossed helical gearing.

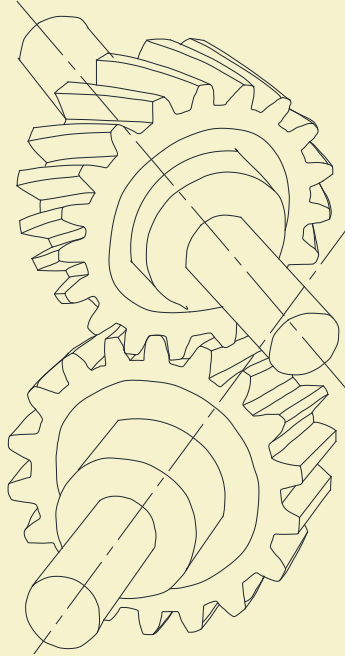
Worm Gearing consists of Worm and Worm Wheel.

Unlike 90° crossed helical gearing the worm (pinion) usually has **not** more than three teeth, which are in continuous thread form.



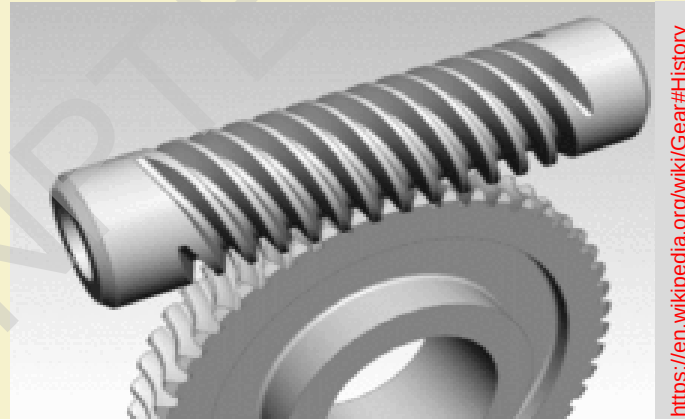
Worm and Worm Wheel in Action

Worm Gear Design (Worm and Worm Wheel) (contd....) :



3D View of Helical gears with non parallel shafts (Crossed Helical Gear)

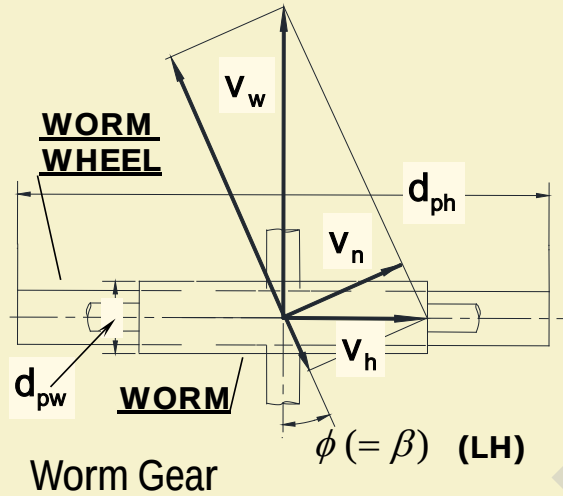
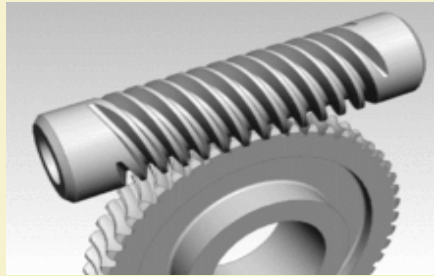
In case Worm the number of teeth is called as number of 'start' (of thread). The cross section of a thread along the normal direction is an involute tooth. The worm wheel is a helical gear. The no. of start may even be one. In that case the drive is irreversible (possesses self locking property).



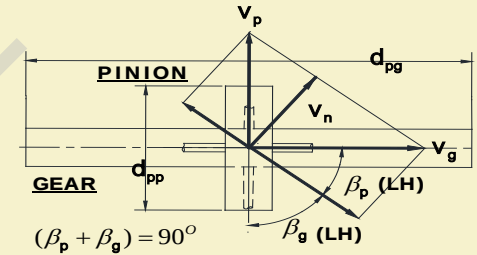
Worm and Worm Wheel in Action

<https://en.wikipedia.org/wiki/Gear#History>

Worm Gear Design (Worm and Worm Wheel) (contd....) :



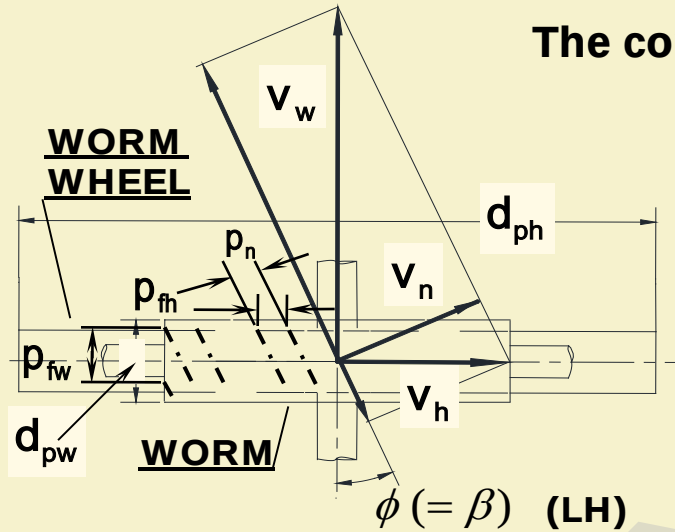
Comparing the helix angles in Worm Gearing with those in 90 degree crossed helical gearing, the helix angle of worm wheel (gear) β_g is low. Let it be β .



Helical gears with 90° shafts.

In case of worm the helix angle β_p (of pinion as in 90° crossed helical gearing) is replaced by $\phi (= 90^\circ - \beta_p) = \beta$ as the lead angle of worm thread. It is shown in the illustrations.

Worm Gear Design (Worm and Worm Wheel) (contd....) :



Worm & Worm Wheel (Gearing)

The constitutive relations in worm gearing are as follows:

The circular 'face' pitches of the worm (p_{fw}) and the worm wheel (p_{fh}) can be expressed in terms of normal circular pitch (p_n) , as :

$$p_n = p_{fh} \cos \phi = p_{fw} \sin \phi$$

The pitch circle diameter of worm, $d_{pw} = \frac{Z_w p_n}{\pi \sin \phi}$

The pitch circle diameter of worm Wheel, $d_{ph} = \frac{Z_h p_n}{\pi \cos \phi}$

Where Z_w is the number of start in worm and Z_h is the number of teeth in worm wheel.

Worm Gear Design (Worm and Worm Wheel) (contd....) :

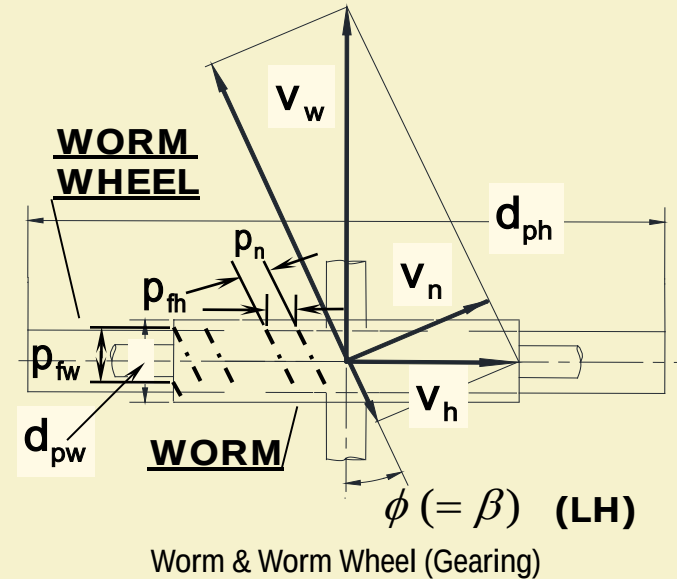
The Centre Distance (CD):

$$A_w = (d_{pw} + d_{ph})/2 = \frac{p_n}{2\pi} \left[\frac{Z_w}{\sin \phi} + \frac{Z_h}{\cos \phi} \right]$$

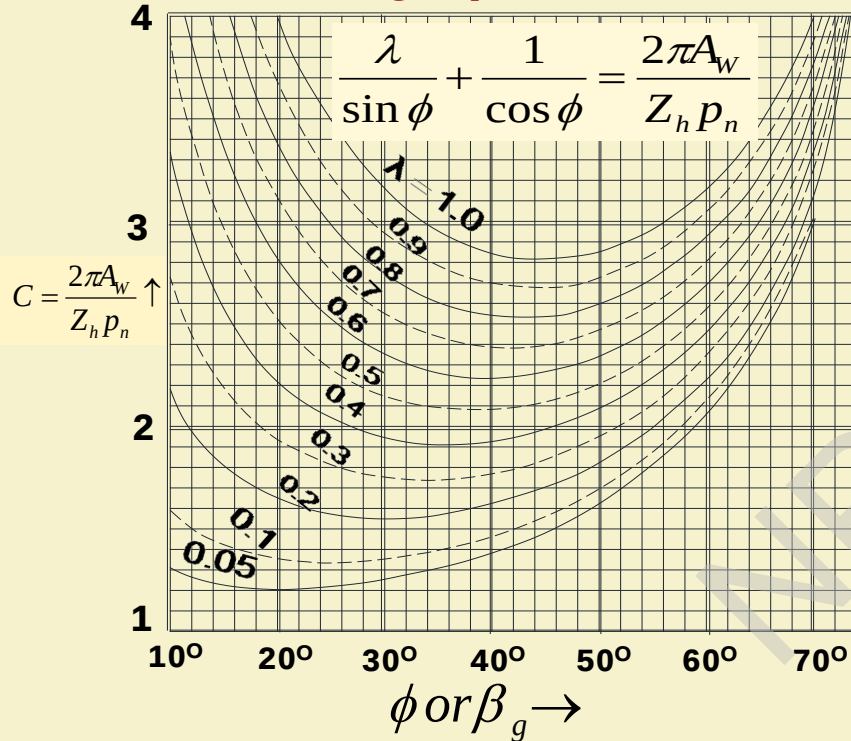
This boils down to: $\frac{\lambda}{\sin \phi} + \frac{1}{\cos \phi} = \frac{2\pi A_w}{p_n Z_h}$

In which $\lambda = Z_w / Z_h$

The expression is same as in crossed helical gear.

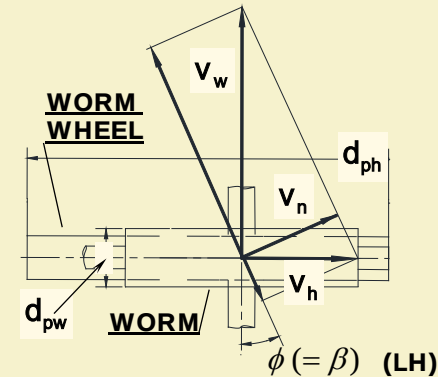


Worm Gear Design (Worm and Worm Wheel) (contd....) :



Design Chart for Worm Gear
(and Helical gears with 90° shafts).

The same design chart, used for crossed helical gears, can also be used for worm gears to choose the initial value of ϕ , the helix angle of the worm wheel teeth and the lead angle of worm tooth.



Worm & Worm Wheel (Gearing)

Worm Gear Design (Worm and Worm Wheel) (contd....) :

Some useful relations:

$$\sin^2 \phi + \cos^2 \phi = 1, \quad \text{Therefore,}$$

$$\text{Rearranging, } \frac{d_{pw}^2}{Z_w^2} + \frac{d_{ph}^2}{Z_h^2} = \frac{p_n^2}{\pi^2}$$

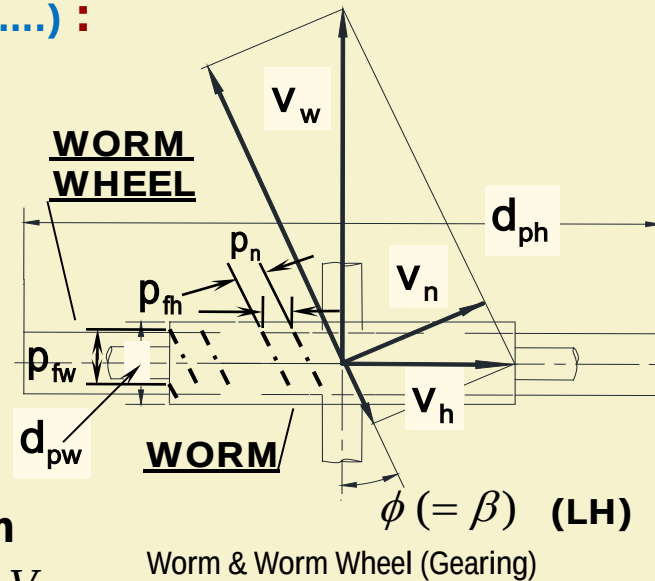
Now let the lead of the worm is l_w .

$$\text{Therefore, } l_w = p_{fh} \times Z_w.$$

N_w and N_h being the speeds in rpm of worm and worm wheel respectively the pitch line velocities of the worm V_w and the worm wheel V_h , which are expressed as:

$$V_w = \pi d_{pw} N_w = N_w Z_w p_{fw}, \quad \text{and,} \quad V_h = \pi d_{ph} N_h = N_h Z_h p_{fh}.$$

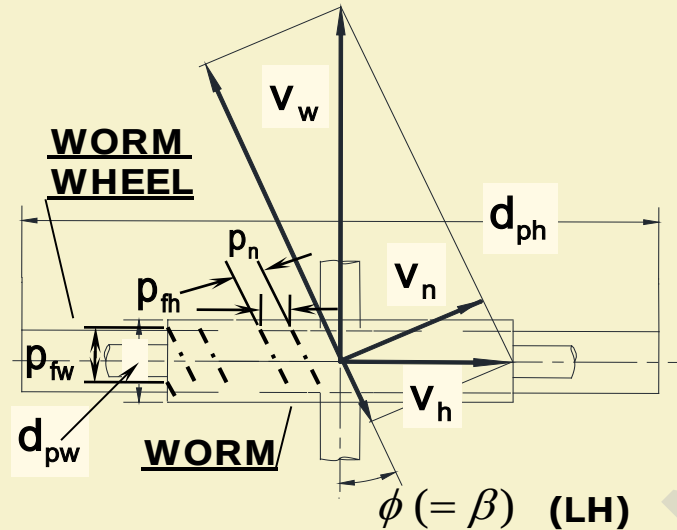
$$\text{Again, } N_h Z_h p_{fh} = N_w Z_w p_{fh} = N_w l_w,$$



$$\text{Therefore, } V_h = N_w l_w.$$

Worm Gear Design (Worm and Worm Wheel) (contd....) :

Some useful relations (contd....) :



Worm & Worm Wheel (Gearing)

$$\text{Now, } \tan \phi = \frac{l_w}{\pi d_w} = \frac{p_{fh}}{p_{fw}} = \frac{V_h}{V_w}$$

It is to be noted that if $\tan \phi$ is less than μ , the coefficient of friction between the worm and worm wheel materials then drive is irreversible.

It is common in single start worm reduction units.

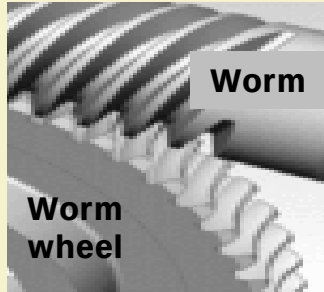
Naturally efficiency of worm gearing is poor.

The worm wheel is generally made of Phosphorus Bronze (Rim fitted on steel or CI) and the teeth is generated by standard hob cutter.

However, generating of worm wheel teeth is done in a little different way towards the final cut, which can be revealed from the view of worm wheel teeth.

Worm Gear Design (Worm and Worm Wheel) (contd....) :

This is done, to have more length of contact of a worm tooth with its mating worm wheel teeth.



It is convenient to determine the worm size after selecting the worm wheel size which can be cut with a standard hob cutter.

Worm teeth can be cut in a lathe with a special turning tool and a fixture.

Beam / Bending strength wise Worm Wheel tooth is weaker than the worm tooth. However, worm wheel is designed in the same way as in helical gear.

Worm Wheel tooth number is usually not below 29. The pressure angle is 20° to 25°. The face width is taken as:

$$b_h = \frac{1}{3}(A_w)^{0.875}$$

Material of Wheel Teeth	K (MPa)
CI or Steel	0.35
Manganese Bronze	0.56
Phosphorous Bronze	0.70
Bakelite	0.875

Factor K in Wear Load Capacity (Steel Worm)

Workout Example :

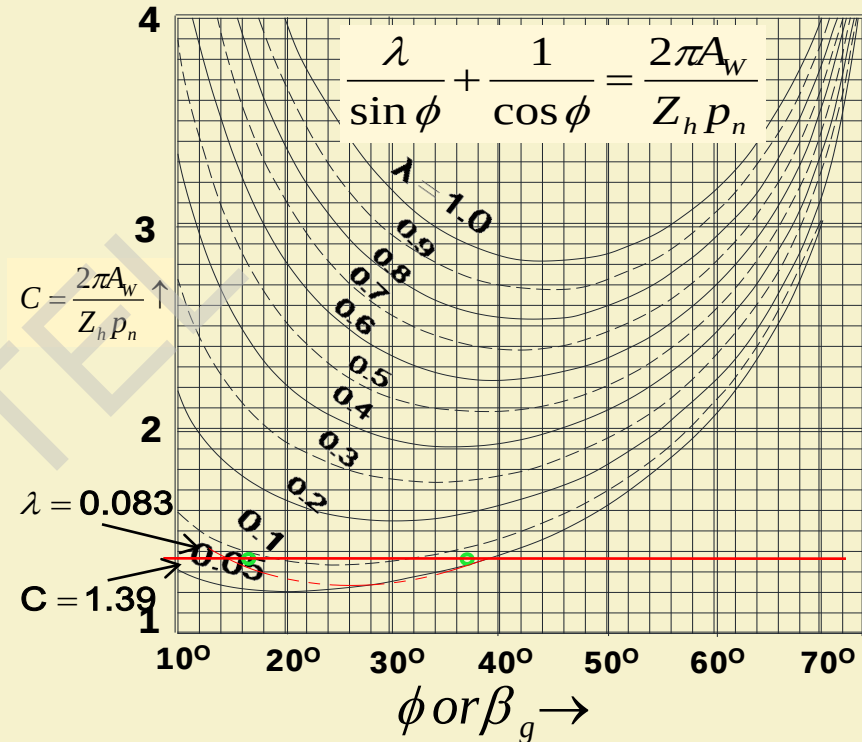
Problem : The centre distance A_w is 200 mm. The normal pitch p_n is 20.42 mm. The worm is quadruple start with the wheel having 48 teeth. Find pcds and helix angle of worm wheel teeth.

Solution :

$$\lambda = Z_w / Z_h = 4 / 48 = 0.083,$$

and
$$C = \frac{2\pi A_w}{Z_h p_n} = \frac{2 \times 200}{48 \times 6} = 1.39$$

Therefore, initial value of $\phi = 15^\circ$ or 36°



Design Chart for Worm Gear
(and Helical gears with 90° shafts).